$$I = \int_a^b f(x)\,dx =$$

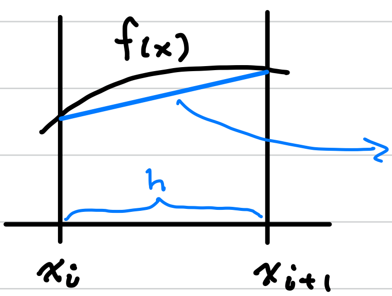
$$=$$

$$=$$

(each point, except first and last, is involved with two trapezoids)

$$I_T = \sum_{i=1}^{N} f(x_i)\,\omega_i \quad , \quad \{\omega_i\} =$$

What is the truncation error?



eq^{n} of top of trapezoid

$$f_t =$$

expand f_{i+1} in Taylor series

$$f_t =$$

$$=$$

$$\int_{x_i}^{x_{i+1}} f_t\,dx =$$

Compare with integral of $f(x)$

$$\int_{x_i}^{x_{i+1}} f\, dx =$$

h^3 term using f_t differs from result based on Taylor series expansion of f

$\therefore$ Truncation error in one step

Error over N steps is

What about roundoff error?

At each step

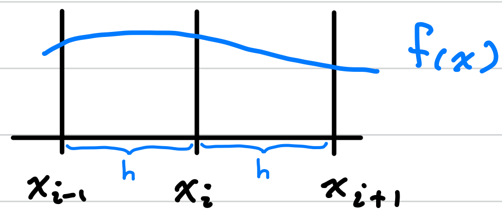
so it turns out

For $\epsilon_M \sim 10^{-16}$ and

smallest error occurs when

Simpson's Rule

Consider a portion of an integral from



Approximate $f(x)$ with a

$f(x) =$

$$I = \int_{x_{i-1}}^{x_{i+1}} \Big[f_i +$$

$$= (x_{i+1} - x_{i-1}) f_i +$$

$$= 2h f_i$$

rewrite $f_i'' = \dfrac{f_{i-1} - 2f_i + f_{i+1}}{h^2} + O(h^2)$

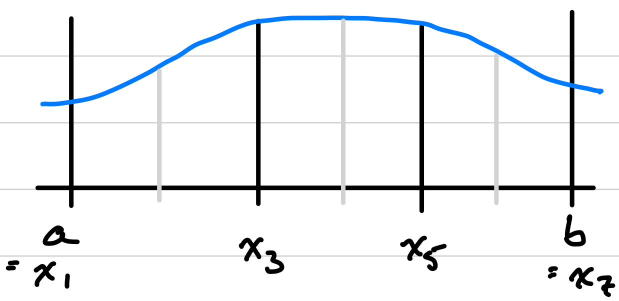
$$= 2h f_i$$

$$=$$

Note: This is equivalent to fitting a quadratic function through (x_{i-1}, f_{i-1}), (x_i, f_i) and (x_{i+1}, f_{i+1}) (cubic term contributes zero to the integral).

Break up entire integral into

Break up entire integral into an



N is

N must

$$I_s = \frac{h}{3}\left(f_1 + 4f_2 + f_3\right)$$

$$= \frac{h}{3}\Big[f_1 + 4f_2$$

truncation error is

$N \sim$ so ϵ_{trunc} is

In terms of general form for numerical integration

$$I_s =$$

Note: consider $N = 3$

$$\{ \omega_i \} =$$

$$\sum_{i=1}^{N} \omega_i =$$

This is generally true for all integration algorithms

$$\sum_{i=1}^{N} \omega_i =$$

Best error, optimal h:

as always $\quad \epsilon_{ro} \approx$

(again, for $b-a \sim f \sim etc \sim 1$)

$$\frac{\epsilon_M}{\sqrt{h}} \approx$$

$$\epsilon_M \approx$$

$$h_{optimal} \approx$$

$$\epsilon_{best} \approx$$

Method of Undetermined Coefficients

Consider writing $f(x)$ as

$f(x) \approx$ on a small interval

General integration formula

$$\int_{\tilde{a}}^{\tilde{b}} f(x)\,dx =$$ with $i = 1, 2,$

becomes

$$\int_{\tilde{a}}^{\tilde{b}} \sum_{n=0}^{p} c_n x^n \, dx =$$

$\rightarrow$ $$\int_{\tilde{a}}^{\tilde{b}} dx\, x^n =$$ for each $n \in 0, \dots p$

$\rightarrow$

$n = 0 \quad \tilde{b} - \tilde{a} \quad =$

$n = 1 \quad \dfrac{\tilde{b}^2 - \tilde{a}^2}{2} \quad =$

$n = 2 \quad \dfrac{\tilde{b}^3 - \tilde{a}^3}{3} \quad =$

$\vdots$

$n = \qquad\qquad =$

or, in matrix form

$$\begin{pmatrix} 1 & 1 & \cdots & 1 \\ x_1 & & & \\ & & & \\ & & & \end{pmatrix} \begin{pmatrix} \omega_1 \\ \\ \\ \\ \end{pmatrix} = \begin{pmatrix} \tilde{b} - \tilde{a} \\ \\ \\ \\ \end{pmatrix}$$

solve for ω_i'

- gives integration formula on fitting $f(x)$ with a polynomial of degree

In terms of dividing up domain of integration, can let

$\tilde{b} - \tilde{a} = \qquad , \quad h =$

E.g. For Simpson's rule,