Gaussian Integration

In Gaussian integration, we fit $f(x)$ over the

by a complete set of

polynomials

$$f(x) =$$

Best to use polynomials like the

, which satisfy

To change limits of integration, can apply

$$x \to \frac{b-a}{2} x + \frac{b+a}{2}$$

$$\int_a^b f(x)\,dx =$$

Assume $f(x)$ is well approximated by a polynomial of order

$$f(x) \simeq$$

For $I = \int_{-1}^{1} f(x)\,dx \simeq$

we need to find

Write $P_{2n-1}(x) =$

e.g. $P_3(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3$

$$p_{2n-1} = p_{n-1} P_n + r_{n-1}$$

express $p_{n-1} =$

$$I = \int_{-1}^{1} dx\, f(x) \simeq$$

$$=$$

$$\therefore \quad I =$$

Let the n zeros of $P_n(x)$ be

Then, at these zeros,

$$p_{2n-1}(x_i) =$$

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Now express $r_{n-1}(x) =$ (in basis of Legendre polynomials)

so $p_{2n-1}(x_i) = r_{n-1}(x_i) =$

where x_i

e.g. $n = 3$ $(2n-1 = 5,\ k_{max} = \qquad)$

$p_{2n-1}(x_1) = p_5(x_1) =$

3 eq$^{\underline{n}}$s, 3 unknowns

or in general

$$\begin{pmatrix} p_{2n-1}(x_1) \\ \\ \\ \end{pmatrix} = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}\begin{pmatrix} \\ \\ \\ \end{pmatrix}$$

$\overleftrightarrow{P}$

$\overleftrightarrow{P}^{-1}$

Then $\begin{pmatrix} \\ \\ \end{pmatrix} = \begin{pmatrix} \\ \\ \end{pmatrix}$

or $\tilde{\omega}_k =$

back to integral

$I = \int_{-1}^{1} f(x)\,dx \doteq$

multiply by I =

$I =$

$I =$

see Mathematica example, K and G handout