The midpoint method is an example of a

↳ use function calls to

→ need to find

To obtain R-K scheme for

write $y(t+h) =$

$$y'(t) = \frac{dy}{dt} =$$

$$\frac{d^2y}{dt^2} =$$

$$\frac{d^3y}{dt} =$$

$y(t+h) =$

Can also write

☆ $y(t+h) =$

where $c_1 =$

$c_2 =$

$c_3 =$

$c_m =$

Example $m = 2$ (2^{nd} order RK)

- keep terms

$y(t+h) =$ (Taylor)

☆ $y(t+h) =$

$c_1 =$

$c_2 =$

so $y(t+h) =$

$=$

compare with Taylor series

- freedom to choose some parameters

Let's choose $\alpha_1 = 0$ $\rightarrow$

then $y_{i+1} =$

This is the Midpoint method

$c_1 =$

$c_2 =$

or, e.g. choose $\alpha_1 = \frac{1}{2}$ $\rightarrow$

$y_{i+1} =$

$c_1 =$

$c_2 =$

rewrite as

$\tilde{y}_{i+1} =$

"Predictor Corrector Method"

$y_{i+1} =$

Common algorithm based on "the" RK method, "Classical RK"

$y(t+h) =$

$c_1 =$

$c_2 =$

$c_3 =$

$c_4 =$

As with RK2,

Adaptive Time Step

- Algorithm chooses time step to give
- can significantly

→ at each step, make sure
→ h needs to be

Simplest case:
- want to step from t to $t+h$ in one jump:

and two jumps:

Error in y^*_{n+1} is $\simeq$

Error in y_{n+1} is $\simeq$

so $\Delta \equiv$

is a good estimate of the error.

Now stipulate that we want the
to be smaller than a

If , accept
→ y(t+h) =

or, we might get a slightly better estimate by
setting since Δ is an
estimate of
(This will reduce the truncation error per step to $O(h^3)$.)

Should also consider , either
by or by increasing by a
factor involving

If , we need to and try again.

option 1

option 2

Simple example

$$\frac{dy}{dx} = \cos(x)$$

Note: RHS does not depend on y.

$y(0)$ on $x \in [0, 2\pi]$

The same approach can be applied to

$y(x+2h) =$ full step of

$y(x+2h) =$ two steps each of

$(c \sim \quad)$ $\Delta =$

$=$

$\Delta \; \alpha$

If $\Delta < \epsilon$, set

or

set

for a possible improvement (assuming)

and make

If $\Delta > \epsilon$,

If the current step size

want to produce an error of

expect $h_{new} =$, so set ,

where is a "safety factor."

How "expensive" is this variable step size approach?

Number of function evaluations is

→ actually only

"cost" of this variable step size approach is

since we could get the better accuracy of

, which takes