Partial Differential Equations - PDEs Three main types of PDEs in physics 1) Elliptic - static problems (7², no time) e.g. -generally, boundary conditions given around 2) Parabolic (V² and ³/_{0t}) e.g. 3) Hyperbolic - propagation (P2 and 2); but not always) e.g. Note: Schrödinger's egcan be viewed as a diffusion eq " with For a general form $\alpha \frac{\partial^2 V}{\partial x^2} + b \frac{\partial^2 V}{\partial x \partial y} + c \frac{\partial^2 V}{\partial y^2} + d \frac{\partial V}{\partial x} + e \frac{\partial V}{\partial y} + f V + g = 0$

These are examples of -linear in dependent variable Another important eg ? is the applied to a small volume of fluid (for incompressible fluid) τ́n-Pp -- the (diffusive) term gives a character. If this term is negligible, the eq= is more Also important in fluids is the The different types of PDEs have different We will consider the different approaches in the next lectures. Do you need a numerical solver?

Separation of Variables
Simplifying PDE analytically can make applying numerical algorithms
easier.
E.g. 1-D Wave
$$eq^{\pm} = \frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial u}{\partial x^2} = 0$$

string with ends fixed at $x = 0$ and $x = L$
 $c - wave speed \sqrt{\frac{1}{p}}, T - tension, p = \frac{M}{L}$
B.c.'s $u(x=0,t) = 0 = u(x=L,t)$
assume $u(x,t) = y(x)f(t)$ -doesn't always work
plug into PDE ...
 $\Rightarrow y(x) = A \sin(kx) + B \cos(kx) - B = 0$.
 $k = k_n = \frac{nT}{L}, n = 1,2,...$
 $f(t) = C \sin(\omega_n t) + D \cos(\omega_n t) - \sin(k_n x)$
 $a_n = \frac{2}{\omega_n L} \int_0^L v_0(x) \sin(k_n x) dx$
 $b_n = \frac{2}{L} \int_0^L u_0(x) \sin(k_n x) dx$

Problem of solving PDE reduces to evaluating definite integrals numerically, unless Uo(x) and Jo(x) are simple. Otherwise, if eqⁿ not separable, must discretize PDE.

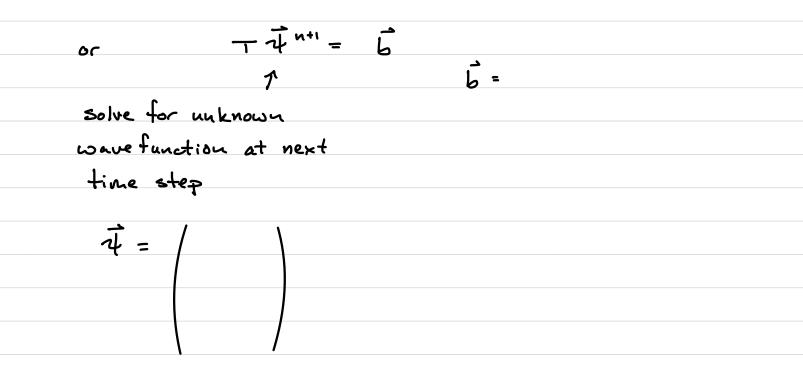
Example: Time - Dependent Schrödinger Equation

$$H + (F,t) = it \frac{\partial}{\partial t} + (F,t)$$

 $- \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$
 $-\frac{1}{2}$
 $-\frac{1}{2}$

for finite difference Instead, use expression -iHst ~ C which is H Hermitian , P⁺ - $H = H^{+}$ so we get $\mathcal{A}_{j}^{n+1} =$ 00 H4; = (1+ + Hist) +; "=

Similarly (1-1/2 Hist) 4;" 2 call Τ= _*= get



Boundary conditions imply