In[1]:= **Remove["Global`*"]**

Wave equation solution

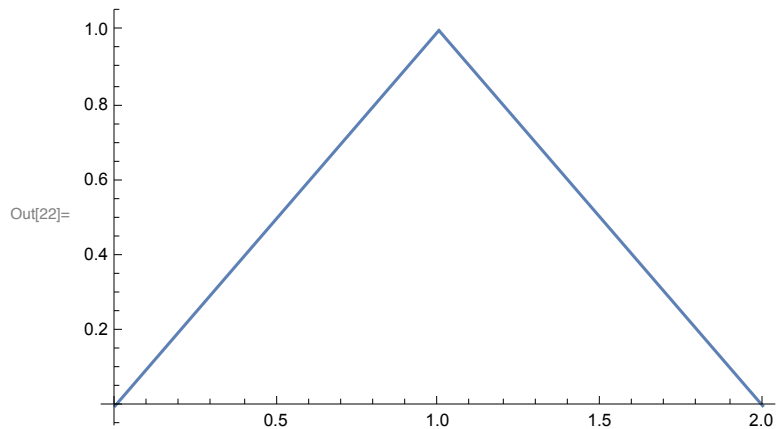See notes for details

In[19]:= **h = 1;**
L = 2;
y0[x_] := Piecewise[{{2 $\frac{h}{L}$ x, x < $\frac{L}{2}$}, {2 $\frac{h}{L}$ (L - x), x ≥ $\frac{L}{2}$}}]

In[22]:= **Plot[y0[x], {x, 0, L}]**

Out[22]=

In[23]:= **Nterms = 3**

Out[23]= 3

In[24]:= **k = n $\frac{\pi}{L}$**

Out[24]= $\frac{n\pi}{2}$

In[25]:= **b = Table[$\frac{2}{L}$ Integrate[y0[x] Sin[k x], {x, 0, L}], {n, 1, Nterms}]**

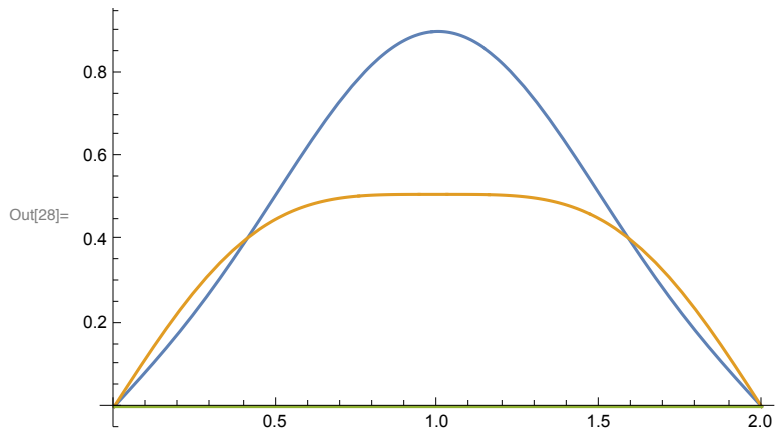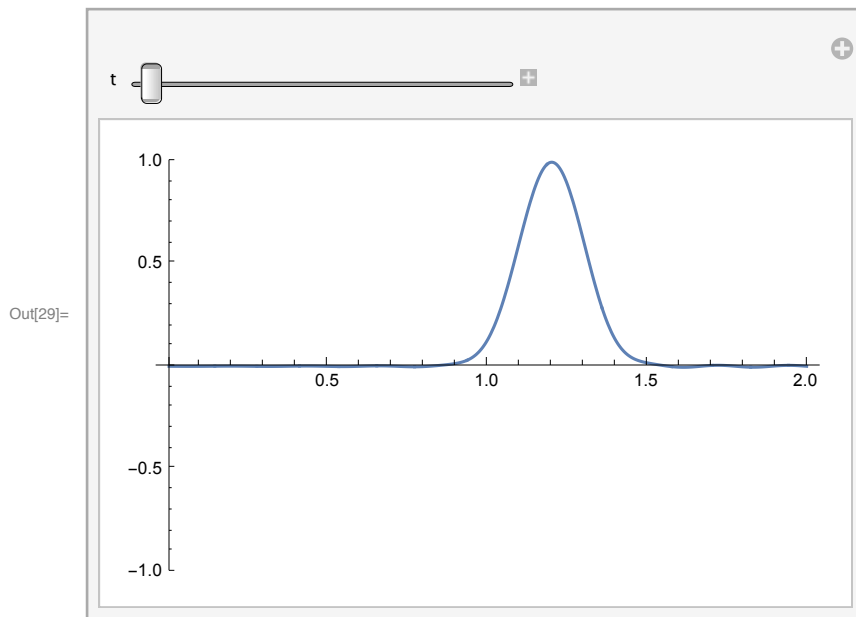Out[25]= $\left\{\frac{8}{\pi^2}, 0, -\frac{8}{9\pi^2}\right\}$

In[26]:= **v = 1;**
y[x_, t_] = Sum[b[[n]] Sin[k x] Cos[k v t], {n, 1, Nterms}]

Out[27]= $\frac{8 \cos\left[\frac{\pi t}{2}\right] \sin\left[\frac{\pi x}{2}\right]}{\pi^2} - \frac{8 \cos\left[\frac{3\pi t}{2}\right] \sin\left[\frac{3\pi x}{2}\right]}{9\pi^2}$

In[28]:= `Plot[{y[x, 0], y[x, 0.5], y[x, 1]}, {x, 0, L}]`

Out[28]=

In[29]:= `Manipulate[Plot[y[x, t], {x, 0, L}, PlotRange → {All, {-1, 1}}], {t, 0, 20}]`

Out[29]=

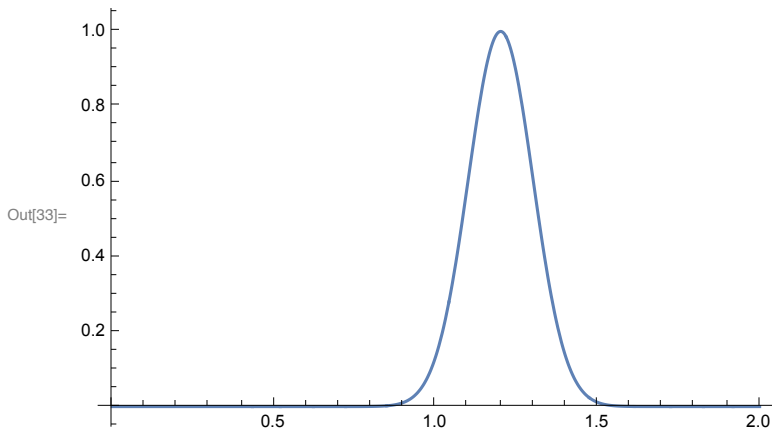

Wave equation solution - different initial conditions

See notes for details

In[30]:= `h = 1;`
`L = 2;`
`y0[x_] := Exp[`$\frac{-(x-1.2)^2}{0.02}$`]`

In[33]:= `Plot[y0[x], {x, 0, L}, PlotRange → All]`

Out[33]=

In[34]:= `Nterms = 17`

Out[34]= 17

In[35]:= $k = n \dfrac{\pi}{L}$

Out[35]= $\dfrac{n\,\pi}{2}$

In[36]:= `b = Table[`$\dfrac{2}{L}$`NIntegrate[y0[x] Sin[k x], {x, 0, L}], {n, 1, Nterms}]`

NIntegrate::ncvb :

NIntegrate failed to converge to prescribed accuracy after 9 recursive bisections in x near {x} = {1.17966}. NIntegrate obtained 1.46150452851046\`*^-16 and 2.8943466025971194\`*^-17 for the integral and error estimates. ≫

NIntegrate::ncvb :

NIntegrate failed to converge to prescribed accuracy after 9 recursive bisections in x near {x} = {1.17966}. NIntegrate obtained -1.249×10^{-16} and 4.099754604575358\`*^-17 for the integral and error estimates. ≫

NIntegrate::ncvb :

NIntegrate failed to converge to prescribed accuracy after 9 recursive bisections in x near {x} = {1.21091}. NIntegrate obtained 9.996344030316351\`*^-17 and 2.6261171268655912\`*^-17 for the integral and error estimates. ≫

General::stop : Further output of NIntegrate::ncvb will be suppressed during this calculation. ≫

Out[36]= $\{0.235472, -0.140242, -0.131852, 0.195691, 1.4615 \times 10^{-16}, -0.152902,$
$0.0804955, 0.0668965, -0.0877621, -1.249 \times 10^{-16}, 0.0535786, -0.0249329,$
$-0.0183157, 0.0212397, 9.99634 \times 10^{-17}, -0.0101315, 0.00416752\}$

In[37]:= `v = 1;`
`y[x_, t_] = Sum[b[[n]] Sin[k x] Cos[k v t], {n, 1, Nterms}]`

Out[38]= $0.235472 \cos\left[\frac{\pi t}{2}\right] \sin\left[\frac{\pi x}{2}\right] - 0.140242 \cos[\pi t] \sin[\pi x] - 0.131852 \cos\left[\frac{3 \pi t}{2}\right] \sin\left[\frac{3 \pi x}{2}\right] +$

$0.195691 \cos[2 \pi t] \sin[2 \pi x] + 1.4615 \times 10^{-16} \cos\left[\frac{5 \pi t}{2}\right] \sin\left[\frac{5 \pi x}{2}\right] -$

$0.152902 \cos[3 \pi t] \sin[3 \pi x] + 0.0804955 \cos\left[\frac{7 \pi t}{2}\right] \sin\left[\frac{7 \pi x}{2}\right] +$

$0.0668965 \cos[4 \pi t] \sin[4 \pi x] - 0.0877621 \cos\left[\frac{9 \pi t}{2}\right] \sin\left[\frac{9 \pi x}{2}\right] -$

$1.249 \times 10^{-16} \cos[5 \pi t] \sin[5 \pi x] + 0.0535786 \cos\left[\frac{11 \pi t}{2}\right] \sin\left[\frac{11 \pi x}{2}\right] -$

$0.0249329 \cos[6 \pi t] \sin[6 \pi x] - 0.0183157 \cos\left[\frac{13 \pi t}{2}\right] \sin\left[\frac{13 \pi x}{2}\right] +$

$0.0212397 \cos[7 \pi t] \sin[7 \pi x] + 9.99634 \times 10^{-17} \cos\left[\frac{15 \pi t}{2}\right] \sin\left[\frac{15 \pi x}{2}\right] -$

$0.0101315 \cos[8 \pi t] \sin[8 \pi x] + 0.00416752 \cos\left[\frac{17 \pi t}{2}\right] \sin\left[\frac{17 \pi x}{2}\right]$

In[39]:= `Plot[{y[x, 0], y[x, 0.5], y[x, 1]}, {x, 0, L}, PlotRange → All]`

Out[39]=
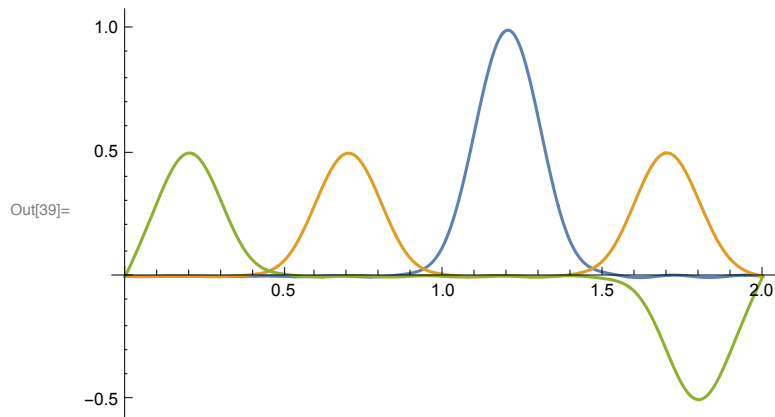
In[40]:= `Manipulate[Plot[y[x, t], {x, 0, L}, PlotRange → {All, {-1, 1}}], {t, 0, 4}]`

Out[40]=