Transverse Waves on Vibrating Strings

Wave speed on a string with tension T and mass per unit length μ:

$$v^2 = \frac{T}{\mu}$$

S-J Problem 13: A telephone cord is 4.0 m long and has a mass of 0.2 kg. A transverse pulse is produced by plucking one end of the taut cord. The pulse makes 4 trips down and back along the cord in 0.8s. What is the tension in the cord ?

Transverse Waves on Vibrating Strings

Wave speed on a string with tension T and mass per unit length μ:

$$v^2 = \frac{T}{\mu}$$

S-J Problem 13: A telephone cord is 4.0 m long and has a mass of 0.2 kg. A transverse pulse is produced by plucking one end of the taut cord. The pulse makes 4 trips down and back along the cord in 0.8s. What is the tension in the cord ?

$$\mu = 0.2\text{kg}/4.0\text{m} = 0.05\,kg/m$$

$$v = 8 * 4.0\text{m}/0.8\text{s} = 40\text{m}/s$$

$$v^2 = 1600\,m^2/s^2$$

$$T = \mu\, v^2 = 80\,kg\,m/s^2 = 80\,N$$

Transverse Waves on Vibrating Strings

Example 13.5: Rescuing a hiker

- *An 80.0 kg hiker is trapped on a mountain ledge...*
- *A helicopter rescues the hiker by hovering above him and lowering a cable to him. The mass of the cable is 8.0 kg, and its length is 15.0 m.*
- *A chair of mass 70.0 kg is attached to the end of the cable.*
- *The hiker attaches himself to the chair and the helicopter accelerates upwards.*
- *The hiker is terrified, and tries to signal the pilot be sending transverse pulses up the cable.*

A pulse takes 0.25s to travel the length of the cable.
What is the acceleration of the helicopter?

Transverse Waves on Vibrating Strings

Example 13.5: Rescuing a hiker

- *An 80.0 kg hiker is trapped on a mountain ledge...*
- *A helicopter rescues the hiker by hovering above him and lowering a cable to him. The mass of the cable is 8.0 kg, and its length is 15.0 m.*
- *A chair of mass 70.0 kg is attached to the end of the cable.*
- *The hiker attaches himself to the chair and the helicopter accelerates upwards.*
- *The hiker is terrified, and tries to signal the pilot be sending transverse pulses up the cable.*

A pulse takes 0.25s to travel the length of the cable.
What is the acceleration of the helicopter?

$$v^2 = \frac{T}{\mu}$$

$$v = 15.0\text{m}/0.25\text{s} = 60\text{m}/s$$

$$\mu = 8.0\text{kg}/15.0\text{m} = 0.53\text{kg}/m$$

$$\Rightarrow T = \mu v^2 = 1920\,N$$

Transverse Waves on Vibrating Strings

Example 13.5: Rescuing a hiker

- *An 80.0 kg hiker is trapped on a mountain ledge...*
- *A helicopter rescues the hiker by hovering above him and lowering a cable to him. The mass of the cable is 8.0 kg, and its length is 15.0 m.*
- *A chair of mass 70.0 kg is attached to the end of the cable.*
- *The hiker attaches himself to the chair and the helicopter accelerates upwards.*
- *The hiker is terrified, and tries to signal the pilot be sending transverse pulses up the cable.*

A pulse takes 0.25s to travel the length of the cable.
What is the acceleration of the helicopter?

$$T = \mu v^2$$
$$= 1920\,N$$

$$T = m_{total}\,g + m_{total}\,a$$
$$a = \frac{T}{m_{total}} - g$$

$$a = \frac{1920}{70+80+8} - 9.8\,m/s^2$$
$$= 2.4\,m/s^2$$

The book ignores the mass of the rope and gets $3\,m/s^2$ *(and thats ok too)*

Reflection and Transmission of Waves

Lets look back at our movie of the travelling pulse.

Lets assume the support at the attached end is rigid.
As the pulse approaches the rigid support it exerts an upward force on the support.
Newton's third law: the support exerts a reaction force on the string in the downward direction.

Reflection and Transmission of Waves

Travelling wave with a free end.

Lets look at some interactive simulations

http://phet.colorado.edu/simulations/sims.php?sim=Wave_on_a_String

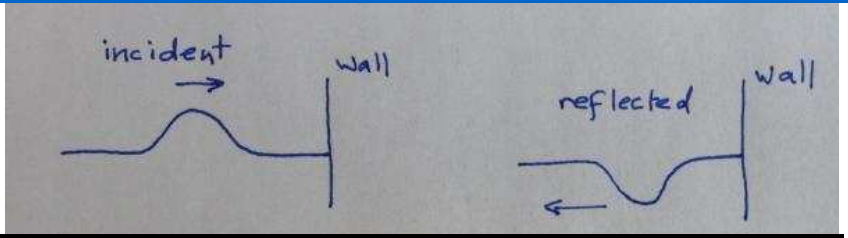
Lets assume the string is attached to a ring that can slide up and down the support.

The ring is then pulled back down by the downward component of string tension.

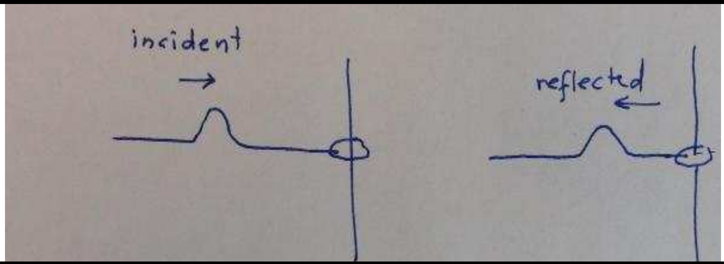
The pulse itself is reflected back without inversion.

Reflection and Transmission of Waves

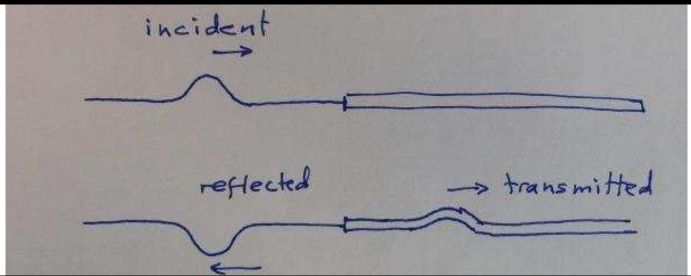
Fixed End
Reflection: inverted
Transmission: none



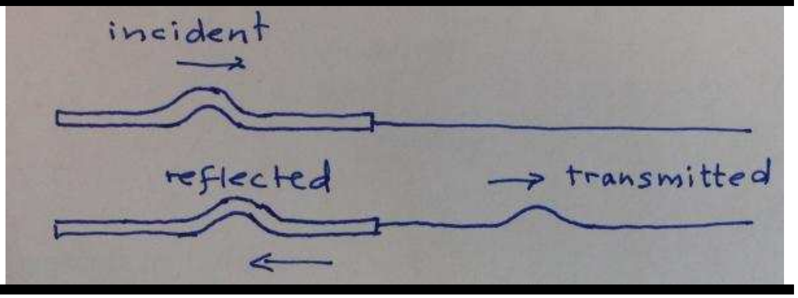
Loose End
Reflection: not inverted
Transmission: none



Pulse encounters denser medium
Reflection: inverted
Transmission: not inverted



Pulse encounters less dense medium
Reflection: not inverted
Transmission: not inverted



Energy Transfer in Sinusoidal Waves

Lets consider the total energy in simple harmonic motion (SHM).

$$E = K(t) + U(t) = \frac{1}{2} k A^2 = \frac{1}{2} m \omega^2 A^2$$

While kinetic energy and potential energy change with time, the total energy is fixed (provided there is no damping)

For a travelling wave, as a wave pass an infinitesimal element of mass dm = μ dx, that element moves in SHM

$$dE = \frac{1}{2} \mu \omega^2 A^2 dx$$

$$E_\lambda = \int_0^\lambda dE = \frac{1}{2} \mu \omega^2 A^2 \int_0^\lambda dx = \frac{1}{2} \mu \omega^2 A^2 \lambda$$

Energy Transfer in Sinusoidal Waves

For a travelling wave, as a wave pass an infinitesimal element of mass dm = μ dx, that element moves in SHM

$$dE = \frac{1}{2}\mu\omega^2 A^2 dx$$

$$E_\lambda = \int_0^\lambda dE = \frac{1}{2}\mu\omega^2 A^2 \int_0^\lambda dx = \frac{1}{2}\mu\omega^2 A^2 \lambda$$

The power or rate of energy transfer is

$$P = \frac{E_\lambda}{T} = \frac{1}{2}\mu\omega^2 A^2 \frac{\lambda}{T} = \frac{1}{2}\mu\omega^2 A^2 v$$

Energy Transfer in Sinusoidal Waves

Example S-J 13.6: A string has a linear mass density μ = 0.05 kg/ m, and is under a tension of 80.0 N. How much power must be supplied to the string to generate sinusoidal waves at a frequency of 60.0 Hz and an amplitude of 6.0 cm ?

The power or rate of energy transfer is

$$P = \frac{E_\lambda}{T} = \frac{1}{2}\mu\omega^2 A^2 v$$

Energy Transfer in Sinusoidal Waves

Example S-J 13.6: A string has a linear mass density μ = 0.05 kg/ m, and is under a tension of 80.0 N. How much power must be supplied to the string to generate sinusoidal waves at a frequency of 60.0 Hz and an amplitude of 6.0 cm ?

$$\mu = 0.05\, kg/m$$

$$\omega = 2\pi f = 2\pi * 60\, Hz = 377\, rad/s$$

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{80\text{N}}{0.05\, kg/m}} = 40\text{m}/s$$

The power or rate of energy transfer is

$$P = \frac{E_\lambda}{T} = \frac{1}{2}\mu\omega^2 A^2 v$$

Energy Transfer in Sinusoidal Waves

Example S-J 13.6: A string has a linear mass density μ = 0.05 kg/ m, and is under a tension of 80.0 N. How much power must be supplied to the string to generate sinusoidal waves at a frequency of 60.0 Hz and an amplitude of 6.0 cm ?

$$\mu = 0.05\, kg/m$$

$$\omega = 2\pi f = 2\pi * 60\, Hz = 377\, rad/s$$

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{80\text{N}}{0.05\, kg/m}} = 40\text{m}/s$$

The power or rate of energy transfer is

$$P = \frac{1}{2}\mu\omega^2 A^2 v = 0.5 * 0.05 * (377)^2 * 0.06^2 * 40 = 512\, W$$

Energy Transfer in Sinusoidal Waves

S-J Ch13, Problem 21: Sinusoidal waves 5.0 cm in amplitude are to be transmitted along a string that has a linear mass density of 0.04 kg/m. If the source can deliver a maximum power of 300 W, what is the highest frequency at which the source can operate?

$$\mu = 0.04\, kg/m$$

$$P_{max} = 300\text{W}$$

$$A = 0.05\text{m}$$

We know that:

$$P = \frac{1}{2}\mu\omega^2 A^2 v$$

Energy Transfer in Sinusoidal Waves

S-J Ch13, Problem 21: Sinusoidal waves 5.0 cm in amplitude are to be transmitted along a string that has a linear mass density of 0.04 kg/m. If the source can deliver a maximum power of 300 W, what is the highest frequency at which the source can operate?

$$\mu = 0.04\,kg/m$$
$$P_{max} = 300\mathrm{W}$$
$$A = 0.05\mathrm{m}$$

We know that:

$$P = \frac{1}{2}\mu\omega^2 A^2 v$$

$$\omega_{max} = \sqrt{\frac{2\mathrm{P}_{max}}{\mu A^2 v}}$$

$$f_{max} = \frac{1}{2\pi}\sqrt{\frac{2\mathrm{P}_{max}}{\mu A^2 v}}$$

Energy Transfer in Sinusoidal Waves

S-J Ch13, Problem 21: I forgot to tell you, the string is under a tension of 100 N.

$$\mu = 0.04\, kg/m$$

$$P_{max} = 300\text{W}$$

$$A = 0.05\text{m}$$

$$T = 100\, N$$

We can calculate

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{100}{0.04}} = 50\, m/s$$

$$f_{max} = \frac{1}{2\pi}\sqrt{\frac{2\text{P}_{max}}{\mu A^2 v}}$$

$$\approx 55\, Hz$$

Longitudinal Waves: Sound

Transverse Waves: Oscillations are perpendicular to the propagation direction

Longitudinal Waves: Oscillations are perpendicular to the propagation direction

Examples:

- *Slinky (show demo)*
- *Sound Waves*

Note: All my animations and movies are available at www.physics.mun.ca/~anand/movies