Circle instructor: Poduska or Morrow 1 Name: SOLUTIONS.
Lab period: ____________ Student Number: ____________________

MEMORIAL UNIVERSITY OF NEWFOUNDLAND
DEPARTMENT OF PHYSICS AND PHYSICAL OCEANOGRAPHY

Physics 1051 Winter 2009

Term Test 2 March 20, 2009

INSTRUCTIONS:
1. Do all questions. Marks are indicated in the left margin. Budget time accordingly.
2. Write your name and student number on each page.
3. You may use a calculator. All other aids are prohibited.
4. Write answers neatly in space provided. If necessary, continue onto the back of the page.
5. Do not erase or use “whiteout” to correct answers. Draw a line neatly through material to be replaced and continue with correction.
6. Assume all information given is accurate to 3 significant figures.
7. Don’t panic. If something isn’t clear, ASK!

SEE LAST PAGE FOR SOME POTENTIALLY USEFUL FORMULAE AND CONSTANTS

For office use only:

1	2	3	4	total

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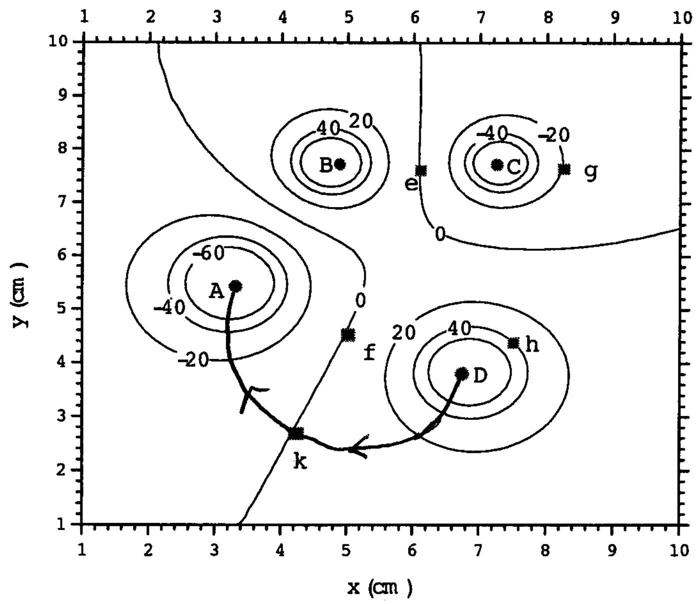
[10] 1. The graph below shows equipotential lines around three point charges; **A**, **B**, **C**, and **D**. The lines are drawn in steps of 20 V for all voltages between -60 V and +60 V. The electric potential is taken to be 0 at infinity.



(a) What is the sign of charge **A**? Which other charge has the same sign?

SIGN OF A: NEGATIVE

CHARGE C ALSO NEGATIVE

(b) Carefully draw the electric field line that starts on one charge, passes through point **k**, and ends on another charge. Be sure that the direction of the field line and the charges on which it starts and ends are clearly shown.

SEE DIAGRAM: ELECTRIC FIELD LINE IS PERPENDICULAR TO EQUIPOTENTIAL LINES AT CROSSINGS.

. ELECTRIC FIELD LINE STARTS ON POSITIVE, ENDS ON NEGATIVE.

(c) Is the magnitude of the electric field larger at point **e** or at point **f**? Briefly explain your choice.

- $|\vec{E}|$ at e larger than $|\vec{E}|$ at f.
- Reason: separation of equipotential lines for given ΔV smaller near e than near f.

$|\vec{E}| = \Delta V/\Delta d$ so smaller Δd implies larger $|\vec{E}|$

(d) Using the scale on the *x*-axis and the equipotentials, estimate the magnitude of the electric field at point **e**.

$$|\vec{E}_e| = \frac{\Delta V}{\Delta d} = \frac{20V - (-20V)}{0.9cm} = 44.4 \frac{V}{cm} = 4440 \, V/m.$$

(e) How much work is done by the electric field as an electron is moved from point **h** to point **g**? (hint: be careful with the sign)

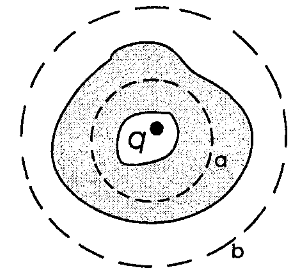
$\Delta U_{electron} = q(V_g - V_h) = -1.6\times10^{-19} C \times (-20V - 40V) = +9.6\times10^{-18} J$

Work by electric field $= -\Delta U = -9.6\times10^{-18}$ J.

To check sign: negative charge going from more positive potential to more negative potential. On average, force due to electric field is opposite displacement.

$\therefore$ from $W = \int_h^g \vec{F}\cdot d\vec{s}$, expect work by electric field to be negative.

[10] 2. (a) A cavity within an uncharged conducting object contains a point charge $q = -5.0\times10^{-15}$ C as shown.



(i) What is the total flux through a spherical closed surface a that is embedded entirely within the conducting material?

"a" entirely in conductor.
WITHIN CONDUCTING MATERIAL, $\vec{E}=0$.
$\therefore \Phi_a = \oint_a \vec{E}\cdot d\vec{A} = 0 \Rightarrow \Phi$ through $a = 0$.

(ii) What is the magnitude of the total flux through a spherical closed surface b that entirely encloses the conducting object?

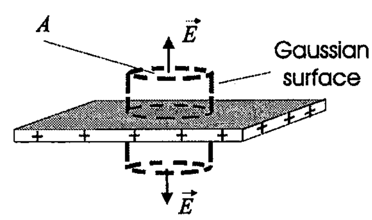
Conductor has no net charge so charge enclosed by $b = -5\times10^{-15}$ C.

$$\therefore |\Phi_b| = \frac{|q_{enclosed}|}{\varepsilon_0} = \frac{|-5\times10^{-15}\,C|}{8.85\times10^{-12}\,\frac{C^2}{N m^2}} = 5.65\times10^{-4}\ Nm^2/C$$

(iii) What is the total charge on the outside surface of the conductor?

For $|\vec{E}|$ at surface a to be zero, charge on inner surface of cavity must be $-q = +5.0\times10^{-15}$ C.
BUT conductor is uncharged, Therefore, charge on outer surface of conductor must be $+q = -5.0\times10^{-15}$ C

(b) The figure to the right shows a portion of an infinite, uniform sheet of charge (positive). A cylindrical Gaussian surface with a cross-sectional area $A = 0.03\ \text{m}^2$ is drawn on the sheet. The electric flux through the ***top*** surface of the cylinder is $15.0\ \text{N}\cdot\text{m}^2/\text{C}$.



(i) What is the magnitude of the electric field at a point 2.0 cm above the surface of the sheet?

$\vec{E}$ outside the charged sheet is uniform. For the end of the Gaussian cylinder above the sheet:

$$\Phi_{TOP} = \vec{E}_{TOP}\times A$$

$$\therefore |\vec{E}| = \frac{15\ Nm^2/C}{0.03\ m^2} = 500\ N/C$$

(ii) What is the total flux through the closed cylindrical Gaussian surface drawn on the figure?

for the Gaussian Cylinder, $\Phi_{TOP} = \Phi_{Bottom}$ ($\vec{E}$ points out at both ends) so

$$\Phi_{TOTAL} = \Phi_{TOP} + \Phi_{BOTTOM} = 2\times15\ Nm^2/C = 30\ N\cdot m^2/C.$$

$(\Phi_{cylinder\ side} = 0)$

(iii) What is the charge per unit area, σ, on the sheet?

$$\Phi_{TOTAL} = \frac{q_{inside}}{\varepsilon_0}$$

$$\therefore 30\ Nm^2/C = \frac{\sigma\times A}{\varepsilon_0}$$

$$\therefore \sigma = 30\ Nm^2/C \times \varepsilon_0 / 0.03\ m^2 = 8.85\times10^{-9}\ C/m^2.$$

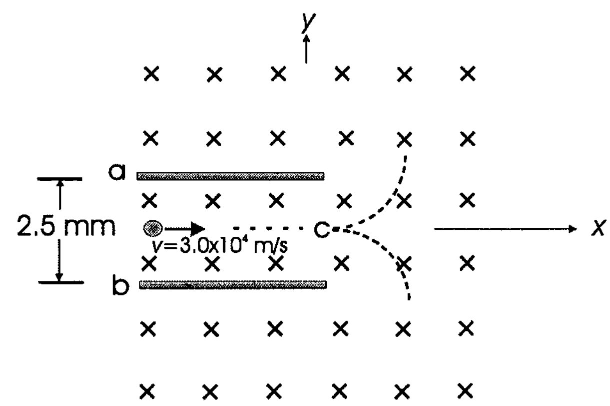
[10] 3. A uniform magnetic field of 13 T is directed into the page as shown. Two oppositely charged plates, **a** and **b**, are separated by 2.5 mm and located so that the uniform electric field between them is perpendicular to the magnetic field. While it is between the charged plates, an ion with a charge of $+e$ (1.6×10^{-19} C), a mass of 1.6×10^{-25} kg and a speed of 3.0×10^{4} m/s travels in a straight line without being deflected.

(a) What is the electric field in the region between the plates? Give your answer in unit vector notation.

(b) What is the potential difference, $V_b - V_a$, between the charged plates?

(c) Beyond point **c** on the ion's path, there is no electric field and its motion is only affected by the uniform magnetic field. What is the magnitude of the centripetal force on the ion after it passes point **c**? Is its path clockwise or counterclockwise?

(d) What is the radius of the ion's path once it is beyond point **c** and affected only by the uniform magnetic field?



(a) for particle to move in a straight line: $\vec{F}_B = -\vec{F}_e$

$\therefore q(\vec{v}\times\vec{B}) = -q\vec{E}$ $\qquad \vec{v} = v\hat{i}$ $\qquad \vec{B} = 13T(-\hat{k})$

$\therefore \vec{E} = -\vec{v}\times\vec{B} = -3\times10^4 \tfrac{m}{s}\,\hat{i} \times (-13T\,\hat{k})$

$= -3.9\times10^5 \tfrac{V}{m}\,\hat{j}$ $\qquad$ (since $\hat{i}\times\hat{k} = -\hat{j}$)

(b) $E_y = -\dfrac{\Delta V}{\Delta y} = -\dfrac{(V_b - V_a)}{(y_b - y_a)} = \dfrac{-(V_b - V_a)}{-2.5\times10^{-3}m}$

$\therefore V_b - V_a = -3.9\times10^5 \tfrac{V}{m} \times 2.5\times10^{-3} m$

$= -975\,V$ $\qquad$ (to get $\vec{E}$ in $-\hat{j}$ direction, $V_b - V_a$ is negative).

OR: $V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{s}$

$= -(3.9\times10^5 \tfrac{V}{m}\hat{j})\cdot(-.0025m\,\hat{j})$

$= -975V$

(c) Centripetal force is provided by magnetic force.

$\therefore |\vec{F}_r| = |F_B| = q|\vec{v}||\vec{B}|$

$= 1.6\times10^{-19}C \times 3\times10^4 \tfrac{m}{s} \times 13T$

$= 6.24\times10^{-14} N.$

(d) $|\vec{F}_r| = \dfrac{mv^2}{r}$

$\therefore r = \dfrac{mv^2}{|\vec{F}_r|} = \dfrac{1.6\times10^{-25}kg \times (3\times10^4 \tfrac{m}{s})^2}{6.24\times10^{-14}N}$

$= 2.3\times10^{-3} m.$

$= 2.3$ mm.

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[10] 4. A uniformly charged ring of radius a is located with its centre at the origin such that the plane of the ring is perpendicular to the x-axis.

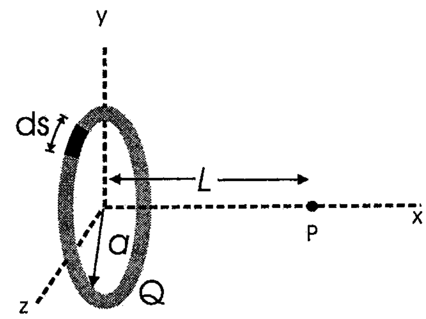
(a) If the total charge on the ring is Q, how much charge, dq, is found on the very small ring segment of length ds, as shown?

(b) What is the contribution to the electric potential at point P from the charge in segment ds? Assume $V = 0$ at infinity.

(c) Starting from $V = k_e \int \frac{dq}{r}$, find an expression for the electric potential at point P due to the entire ring.

(d) What is the electric potential at the origin due to the entire ring?

(e) Assume that the radius of the ring is $a = 2.5\text{ cm}$ and that the total charge on the ring is $Q = 6.0\times10^{-12}\text{ C}$. How much work is done by the electric field if a particle with a charge of $q = -3.2\times10^{-19}\text{ C}$ is moved from the origin to point P at $L = 9.0\text{ cm}$?



(a) $dq = \frac{ds}{2\pi a} \times Q$

(b) For pt. charge, $V = \frac{kq}{r}$

$\therefore dV = \frac{k\,dq}{\sqrt{a^2+L^2}} = \frac{kQ}{2\pi a}\frac{ds}{\sqrt{a^2+L^2}}$

(c) $V = \int_{ring} \frac{k\,dq}{\sqrt{a^2+L^2}} = \frac{kQ}{2\pi a} \times \frac{1}{\sqrt{a^2+L^2}} \times \int_{circumference} ds$

$= \frac{kQ}{2\pi a} \times \frac{1}{\sqrt{a^2+L^2}} \times 2\pi a$

$= \frac{kQ}{\sqrt{a^2+L^2}}$

(d) at origin, $L=0 \Rightarrow V_{origin} = \frac{kQ}{a}$

(e) $\Delta U = q\Delta V = q \times \left[\frac{kQ}{\sqrt{a^2+L^2}} - \frac{kQ}{a}\right]$

$= -3.2\times10^{-19}\,C \times 8.99\times10^{9}\,\frac{Nm^2}{C^2} \times 6\times10^{-12}\,C \times \left[\frac{1}{\sqrt{(.025m)^2+(.09m)^2}} - \frac{1}{.025m}\right]$

$= +5.06\times10^{-19}\,J$

$W_{\text{by electric field}} = -\Delta U = -5.06\times10^{-19}\,J$

(Electric force on negative charge is opposite to displacement from origin to P so W is negative).

Some Potentially Useful Formulae and Constants:

$$\vec{F}_{12} = k_e \frac{q_1 q_2}{r^2}\hat{r}_{12}$$

$$U_{12} = k_e \frac{q_1 q_2}{r_{12}}$$

$$\vec{E} = k_e \frac{q}{r^2}\hat{r}$$

$$\Delta U = -q\int_A^B \vec{E}\cdot d\vec{s}$$

$$\vec{E} = k_e \sum_i \frac{q_i}{r_i^2}\hat{r}_i$$

$$\Delta V = V_B - V_A = -\int_A^B \vec{E}\cdot d\vec{s}$$

$$\vec{E} = k_e \int \frac{dq}{r^2}\hat{r}$$

$$\Delta U = q\Delta V$$

$$\Phi_E = \int \vec{E}\cdot d\vec{A}$$

$$\vec{E} = -\left(\frac{dV}{dx}\hat{i} + \frac{dV}{dy}\hat{j} + \frac{dV}{dz}\hat{k}\right)$$

$$\Phi_E = \frac{q_{\text{inside}}}{\varepsilon_0}$$

$$R = \frac{\Delta V}{I}$$

$$V = k_e \frac{q}{r}$$

$$\vec{F}_B = q\vec{v}\times\vec{B}$$

$$V = k_e \sum_i \frac{q_i}{r_i}$$

$$a_r = \frac{v^2}{r}$$

$$V = k_e \int \frac{dq}{r}$$

$$V_{\text{sphere}} = \frac{4}{3}\pi r^3$$

$C_{\text{circle}} = 2\pi r$ (circumference)

$$A_{\text{sphere}} = 4\pi r^2$$

$$A_{\text{circle}} = \pi r^2$$

Physical constants:

$$k_e = \frac{1}{4\pi\varepsilon_0} = 8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2$$

$$e = 1.602\times 10^{-19}\ \text{C}$$

$$\varepsilon_0 = 8.85\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2$$

$$m_e = 9.11\times 10^{-31}\ \text{kg}$$

Mathematical formulae:

$$\int \frac{dr}{r^2} = -\frac{1}{r}$$

$$\int \frac{dx}{\left(x^2+y^2\right)^{3/2}} = \frac{x}{y^2\sqrt{x^2+y^2}}$$

$$\int \frac{dx}{x} = \ln x$$

$$\int \frac{x\,dx}{\left(x^2+y^2\right)^{3/2}} = -\frac{1}{\sqrt{x^2+y^2}}$$

$$\int \frac{dx}{\sqrt{x^2+y^2}} = \ln\left[x+\sqrt{x^2+y^2}\right]$$

$$\vec{A}\cdot\vec{B} = AB\cos\theta = A_x B_x + A_y B_y + A_z B_z$$

$$\vec{A}\times\vec{B} = \left(A_y B_z - A_z B_y\right)\hat{i} + \left(A_z B_x - A_x B_z\right)\hat{j} + \left(A_x B_y - A_y B_x\right)\hat{k}$$