MEMORIAL UNIVERSITY OF NEWFOUNDLAND
DEPARTMENT OF PHYSICS AND PHYSICAL OCEANOGRAPHY

Physics 1051 Winter 2010

Term Test 2 March 12, 2010

INSTRUCTIONS:
1. Do all questions. Marks are indicated in the left margin. Budget time accordingly.
2. Write your name and student number on each page.
3. You may use a calculator. All other aids are prohibited.
4. Write answers neatly in space provided. If necessary, continue onto the back of the page.
5. Do not erase or use "whiteout" to correct answers. Draw a line neatly through material to be replaced and continue with correction.
6. Assume all information given is accurate to 3 significant figures.
7. Don't panic. If something isn't clear, ASK!

SEE LAST PAGE FOR SOME POTENTIALLY USEFUL FORMULAE AND CONSTANTS

For office use only:

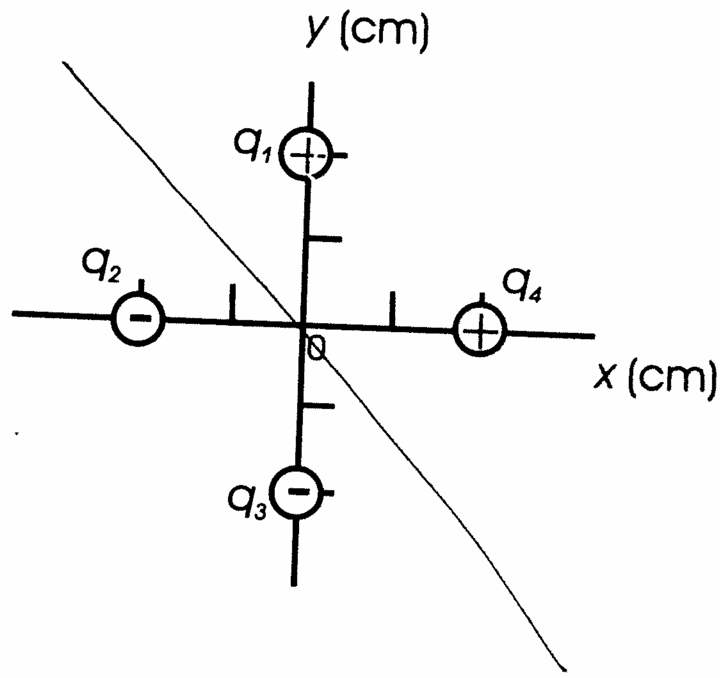
1	2	3	4	total

Circle instructor: Yethiraj or Morrow
Lab period: ___________

Name: ___________
Student Number: ___________

[10] 1. Four charges are positioned on the x-y plane as shown:

$q_1 = +4.5\times10^{-9}$ C is at $x = 0$ cm, $y = +2$ cm.
$q_2 = -4.5\times10^{-9}$ C is at $x = -2$ cm, $y = 0$ cm.
$q_3 = -4.5\times10^{-9}$ C is at $x = 0$ cm, $y = -2$ cm.
$q_4 = +4.5\times10^{-9}$ C is at $x = +2$ cm, $y = 0$ cm.



(a) What is the total electric potential at the origin?

(b) What is the electric field at the origin? Give your answer in unit vector notation.

(c) There is an equipotential line that passes through the origin. Draw it on the figure and briefly justify the line you have drawn.

(d) What is the electric force $\vec{F}_{31}$ on charge q_1 due to charge q_3? Give your answer in unit vector notation.

2 (a) $V = 0$ at $(0,0)$

(b) The fields due to charge 1 and 3 point downwards
2 4 point leftward.

3

$\vec{E}_1 = (-\hat{j})\, k_e \dfrac{q_1}{r^2} = (-\hat{j})\, \dfrac{(8.99\times10^9)(4.5\times10^{-9})}{(2)^2\times10^{-4}} = -1.01\times10^5 \frac{N}{C}(\hat{j})$

$\vec{E}_3 = \vec{E}_1$

$\vec{E}_4 = \vec{E}_2 = -1.01\times10^5\ \hat{i}\ N/C$

$\vec{E} = \sum_{i=1}^{4} \vec{E}_i = -(2.02\times10^5\ \hat{i} + 2.02\times10^5\ \hat{j})\ N/C$

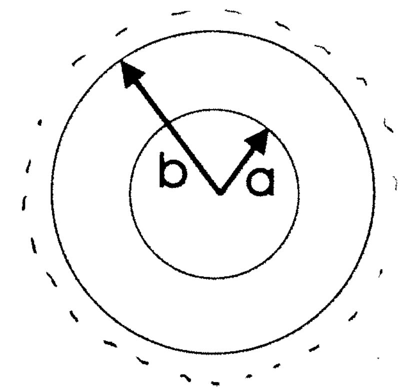
(c) As shown.

2 Every point on line is equidistant to a negative / positive pair
E.g (q_1, q_2) have equal and opposite contributions to the potential.

3 (d) $\vec{F}_{31} = k_e \dfrac{q_1 q_3}{r^2}\hat{r} = \dfrac{(8.99\times10^9)(4.5\times10^{-9})^2}{[.04\,m]^2}\hat{j}$

$= -1.14\times10^{-4}\, N\hat{j}$

[10] 2. (a) A **conducting** spherical shell with an inner radius of $a = 1.5\,\text{cm}$ and an outer radius of $b = 2.0\,\text{cm}$ carries a net charge of $q = -5.0\times10^{-15}\,\text{C}$. The cavity at the centre of the shell is empty.



(i) What is the magnitude of the electric field at a distance $r = 2.5\,\text{cm}$ from the centre of the sphere?

(ii) What is the surface charge density on the inner surface of the shell (i.e. at radius a)? Briefly justify your answer.

(iii) What is the potential difference $\Delta V = V_b - V_a$ between the outer and the inner surface of the sphere? Briefly justify your answer.

(i) Gauss' Law for $r > b$

2

$$\Phi_E = E \cdot 4\pi r^2 = \frac{q}{\epsilon_0} \Rightarrow E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

$$= \frac{(8.99\times10^{9})(-5\times10^{-15})}{(2.5)^2\times10^{-4}}$$

$$\boxed{E = 7.2\times10^{-2}\ \text{N/C}}$$

(ii) Apply Gauss' law for $a < r < b$.

2

Here $E = 0$ $\therefore$

$$\cancel{E}\cdot 4\pi r^2 = Q_{enc}/\epsilon_0 \Rightarrow \boxed{Q_{enc} = 0} \text{ or } Q_{inner} = 0$$

0

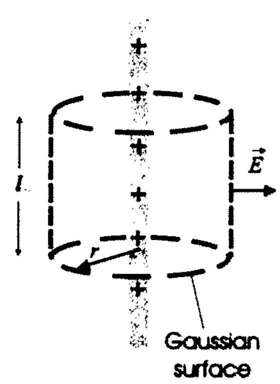
$$\therefore \sigma_{inner} = \frac{Q_{inner}}{4\pi a^2} = 0$$

(iii) Conductor is equipotential $\Rightarrow \Delta V = V_b - V_a = 0$

2

(or) $E_{conductor} = 0 \quad \therefore \quad \Delta V = \int_a^b \vec{E}\cdot d\vec{s} = 0$.

(b) What is the magnitude of the electric field 10.0 cm away from a very long line of charge with a linear charge density of 6.3×10^{-9} C/m? You may find the drawing helpful.



Top and bottom surface $\vec{E}\cdot d\vec{A} = 0$
because $E = E_r\,\hat{r}$ (by symmetry for infinite line of charge)

So only consider the radial wall.

$$\vec{E}\cdot d\vec{A} = E_r\,dA$$

1X

$$\Phi_E = \oint \vec{E}\cdot d\vec{A} = E_r \oint dA = E_r\,(2\pi r l) \quad \text{— (1)}$$

Also $$\Phi_E = \frac{Q_{enc}}{\epsilon_0} = \frac{\lambda l}{\epsilon_0} \quad \text{— (2)}$$

Putting (1) and (2) together

$$\frac{\lambda l}{\epsilon_0} = E_r\,(2\pi r l) \qquad E_r = \frac{\lambda}{2\pi r\epsilon_0}$$

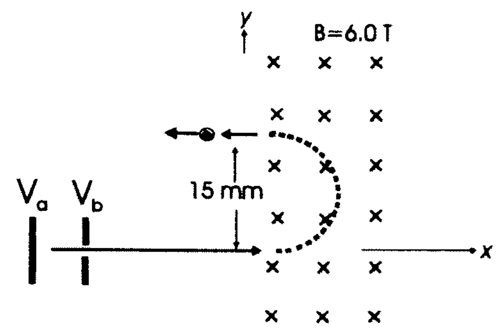
$$= 1133\ \text{N/C}$$

[10] 3. A charged particle starts from rest and is accelerated through a potential difference $\Delta V = V_b - V_a$ as shown. It then enters a region of space containing a uniform magnetic field of 6.0 T directed into the page as shown. The charge travels along a semi-circular path with a diameter of 15.0 mm as shown.

(a) Is the charged particle positive or negative?

(b) If the mass of the particle is $m = 6.64 \times 10^{-27}$ kg and the **magnitude** of its charge is $|q| = 1.6 \times 10^{-19}$ C, what is the speed of the charged particle?

(c) What is the potential difference, $\Delta V = V_b - V_a$, through which the charged particle was initially accelerated?



(a) From right-hand rule:

$\vec{v} \times \vec{B}$ points UPWARDS ~~out of page~~.

3 Clearly $\vec{F}_B = q\vec{v} \times \vec{B}$ points upwards $\therefore$ $q > 0$

(b) $qvB = \frac{mv^2}{r}$

4 $v = \frac{qBr}{m} = \frac{(1.6 \times 10^{-19})(6.0)(0.015)/2}{6.64 \times 10^{-27}}$

$= 0.011 \times 10^8$

$= 1.1 \times 10^6$ m/s

(c) $\Delta U + \Delta K = 0$ (conservation of total energy)

$(\Delta V)q + \frac{1}{2}mv^2 = 0$ (particle starts from rest)

3 $\Delta V = \frac{-\frac{1}{2}mv^2}{q} = \frac{-\frac{1}{2} \times 6.64 \times 10^{-27} \times (1.1)^2 \times 10^{12}}{1.6 \times 10^{-19}}$

$= -2.51 \times 10^4$ V

$\Delta V = V_b - V_a = -2.51 \times 10^4$ V

check ($V_b < V_a$) ✓

[10] 4. A rod of length $L = 15.0$ cm is oriented along the x-axis as shown. A charge $Q = +2\times10^{-6}$ C is uniformly spread along the rod. Point P is located a distance $h = 10.0$ cm above the left end of the rod as shown.

(a) What is the linear charge density, λ, on the rod?

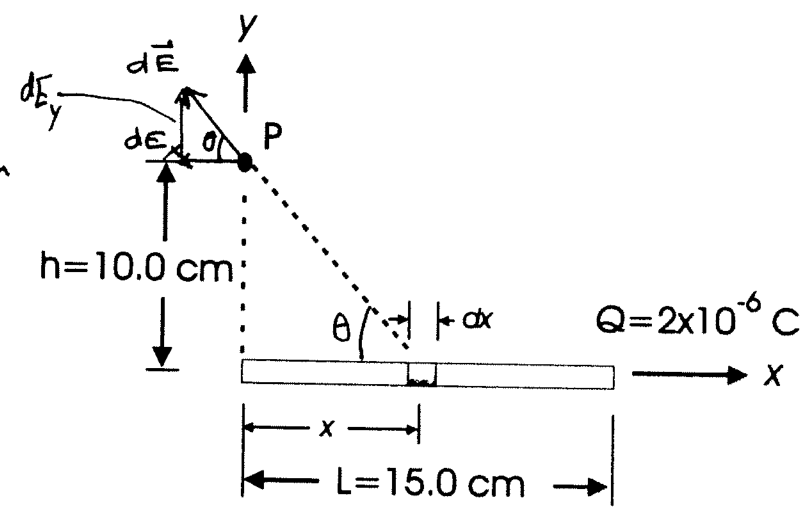
(b) On the diagram, show the direction of the contribution, $d\vec{E}$, to the electric field at P from the charge on segment dx located a distance x from the left end of the rod as shown.

(c) Write an expression for the y-component dE_y of the contribution to the electric field at P from the charge on segment dx.

(d) Calculate the y-component of the electric field, E_y, at a point P due to the entire rod?

2 (a) $\lambda = \dfrac{Q}{L} = \dfrac{2\times10^{-6}}{0.15}$

$= 1.33\times10^{-5}$ C/m



2 (b) $d\vec{E}$ shown on figure

3 (c) $dE_y = |d\vec{E}| \sin\theta$

but $\sin\theta = \dfrac{h}{\sqrt{x^2+h^2}}$

Now $dE \equiv |d\vec{E}|$

$= \dfrac{k_e\, dq}{h^2+x^2} = \dfrac{k_e \lambda \cdot dx}{h^2+x^2}$

$\therefore\ dE_y = dE \sin\theta$

$= k_e \lambda h \dfrac{dx}{(h^2+x^2)^{3/2}}$

3 (d) $E_y = \displaystyle\int_{x=0}^{x=L} dE_y = (k_e\lambda h)\int_{x=0}^{x=L} \frac{dx}{(h^2+x^2)^{3/2}}$

$= (k_e \lambda h)\left\{\left[\dfrac{x}{h^2\sqrt{x^2+h^2}}\right]_{x=L} - 0\right\}$

$= \dfrac{k_e\lambda h L}{h^2\sqrt{L^2+h^2}}$

$= \dfrac{k_e \lambda L}{h\sqrt{L^2+h^2}} = \dfrac{(8.99\times10^{9})(1.33\times10^{-5})(0.15)}{(0.1)\sqrt{(0.1)^2+(0.15)^2}}$

$= 9.95\times10^{5}$ N/C

Some Potentially Useful Formulae and Constants:

$$\vec{F}_{12} = k_e \frac{q_1 q_2}{r^2} \hat{r}_{12}$$

$$U_{12} = k_e \frac{q_1 q_2}{r_{12}}$$

$$\vec{E} = k_e \frac{q}{r^2} \hat{r}$$

$$\Delta U = -q \int_A^B \vec{E} \cdot d\vec{s}$$

$$\vec{E} = k_e \sum_i \frac{q_i}{r_i^2} \hat{r}_i$$

$$\Delta V = V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{s}$$

$$\vec{E} = k_e \int \frac{dq}{r^2} \hat{r}$$

$$\Delta U = q \Delta V$$

$$\Phi_E = \int \vec{E} \cdot d\vec{A}$$

$$\vec{E} = -\left(\frac{dV}{dx} \hat{i} + \frac{dV}{dy} \hat{j} + \frac{dV}{dz} \hat{k} \right)$$

$$\Phi_E = \frac{q_{\text{inside}}}{\varepsilon_0}$$

$$R = \frac{\Delta V}{I}$$

$$V = k_e \frac{q}{r}$$

$$\vec{F}_B = q \vec{v} \times \vec{B}$$

$$V = k_e \sum_i \frac{q_i}{r_i}$$

$$a_r = \frac{v^2}{r}$$

$$V = k_e \int \frac{dq}{r}$$

$$V_{\text{sphere}} = \frac{4}{3} \pi r^3$$

$C_{\text{circle}} = 2\pi r$ (circumference)

$$A_{\text{sphere}} = 4\pi r^2$$

$$A_{\text{circle}} = \pi r^2$$

Physical constants:

$$k_e = \frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9 \ \text{N} \cdot \text{m}^2 / \text{C}^2$$

$$e = 1.602 \times 10^{-19} \ \text{C}$$

$$\varepsilon_0 = 8.85 \times 10^{-12} \ \text{C}^2 / \text{N} \cdot \text{m}^2$$

$$m_e = 9.11 \times 10^{-31} \ \text{kg}$$

Mathematical formulae:

$$\int \frac{dr}{r^2} = -\frac{1}{r}$$

$$\int \frac{dx}{x} = \ln x$$

$$\int \frac{dx}{\sqrt{x^2 + y^2}} = \ln\left[x + \sqrt{x^2 + y^2}\right]$$

$$\int \frac{dx}{\left(x^2 + y^2\right)^{3/2}} = \frac{x}{y^2 \sqrt{x^2 + y^2}}$$

$$\int \frac{x\,dx}{\left(x^2 + y^2\right)^{3/2}} = -\frac{1}{\sqrt{x^2 + y^2}}$$

$$\int \sin\theta \, d\theta = -\cos\theta$$

$$\int \cos\theta \, d\theta = \sin\theta$$

$$\vec{A} \cdot \vec{B} = AB\cos\theta = A_x B_x + A_y B_y + A_z B_z$$

$$\vec{A} \times \vec{B} = \left(A_y B_z - A_z B_y\right)\hat{i} + \left(A_z B_x - A_x B_z\right)\hat{j} + \left(A_x B_y - A_y B_x\right)\hat{k}$$