



**SECTION 1: MULTIPLE CHOICE. (answer in the box provided for each question)**

[2] 1. An electron is traveling with a velocity  $\vec{v} = 550 \text{ m/s } \hat{i}$  when it enters a region of space where the electric field is  $\vec{E} = -1.0 \times 10^{-9} \text{ N/C } \hat{j}$ .

D

- A. After the electron has been in the field for 0.10 s, the  $x$ -component of its velocity is positive and the  $y$ -component of its velocity is negative.
- B. After the electron has been in the field for 0.10 s, the  $x$ -component of its velocity is zero and the  $y$ -component of its velocity is positive.
- C. After the electron has been in the field for 0.10 s, the  $x$ -component of its velocity is positive and the  $y$ -component of its velocity is zero.
- D. After the electron has been in the field for 0.10 s, the  $x$ -component of its velocity is positive and the  $y$ -component of its velocity is positive.

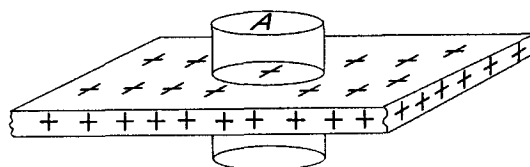
[2] 2. Which of the following statements **IS TRUE**?

B

- A. The surface charge density is uniform over the surface of any charged conductor.
- B. The electric field at the surface of a charged conductor is perpendicular to the surface of the conductor.
- C. The excess charge on a charged conductor is uniformly distributed over its volume.
- D. The equipotentials near the surface of a charged conductor are perpendicular to the surface of the conductor.
- E. The electric field at the surface of a charged conductor is zero.

[2] 3. Imagine a cylinder with cross-sectional area  $A$  drawn through an infinite, uniformly charged insulating slab as shown. The charge per unit area of the slab is  $\sigma$  in  $\text{C/m}^2$ . Which of the following statements **IS TRUE**?

B



- A. The total electric flux through the cylinder ends is 0.
- B. The sum of the electric fluxes through the two cylinder ends is  $\frac{\sigma A}{\epsilon_0}$ .
- C. The sum of the electric fluxes through the two cylinder ends is  $-\frac{\sigma A}{\epsilon_0}$ .
- D. The sum of the fluxes through the two cylinder ends and the curved cylindrical surface is 0.

- [2] 4. The electric potential in a region of space is given by  $V = 7x^2y$  in SI units. Which of the following statements **IS FALSE**?

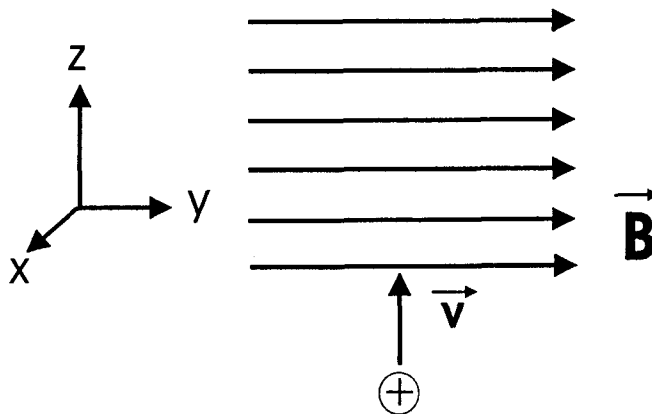
D

- A. The magnitude of the electric field at the origin is 0.
- B. The  $x$ -component of the electric field depends on both  $x$  and  $y$ .
- C. The  $z$ -component of the electric field is 0 everywhere in this region.
- D. The  $y$ -component of the electric field is proportional to  $y$ .

- [2] 5. A positive charge  $+q$  traveling with velocity  $\vec{v} = 357 \text{ m/s } \hat{k}$  enters a region where the magnetic field is  $\vec{B} = 5.75 \text{ T } \hat{j}$  as shown. Which of the following statements **IS TRUE** ?

D

- A. The magnetic force on the charge is in the positive  $x$  direction.
- B. The magnetic force on the charge is in the positive  $y$  direction.
- C. The magnetic force on the charge is in the negative  $y$  direction.
- D. The magnetic force on the charge is in the negative  $x$  direction.



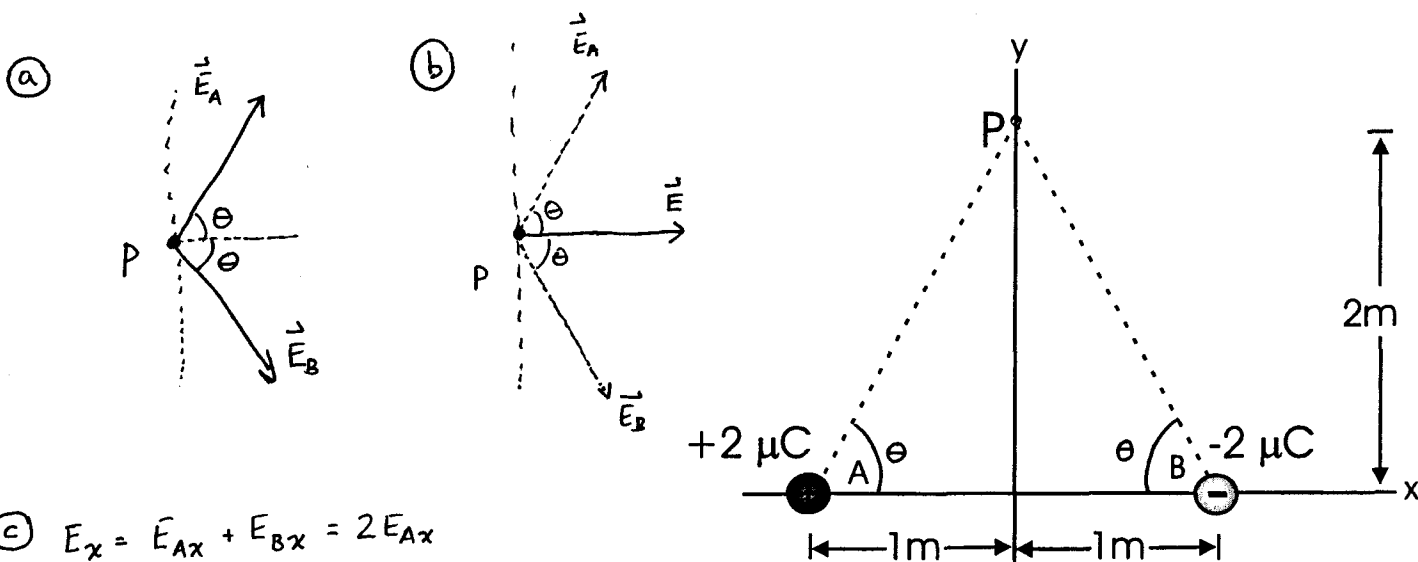
**SECTION 2: DO ALL FOUR (4) QUESTIONS**

[10] 6. A charge of  $+2 \mu\text{C}$  (charge A) is located at  $x = -1.0 \text{ m}$  along the  $x$ -axis. A second charge of  $-2 \mu\text{C}$  (charge B) is located on the  $x$ -axis at  $x = +1.0 \text{ m}$ . The point  $P$  is located on the  $y$ -axis at  $y = 2.0 \text{ m}$ .

[2] (a) Draw vectors representing the electric field at point  $P$  due to charge A ( $\vec{E}_A$ ) and the electric field at point  $P$  due to charge B ( $\vec{E}_B$ ).

[2] (b) Draw the vector representing the total electric field ( $\vec{E}$ ) at point  $P$ .

[6] (c) Find the  $x$ -component and  $y$ -components of the total electric field ( $\vec{E}$ ) at point  $P$ .



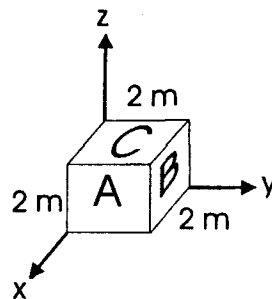
(c)  $E_x = E_{Ax} + E_{Bx} = 2E_{Ax}$

$$E_x = 2 \cdot \frac{kqA}{r^2} \cos \theta = \frac{2 \left( 8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) \left( +2 \times 10^{-6} \text{ C} \right)}{\left( \sqrt{(1\text{m})^2 + (2\text{m})^2} \right)^2} (\cos 63.435^\circ)$$

$E_x = 3220 \text{ N/C}$

$E_y = E_{Ay} + E_{By} = 0 \text{ N/C}$

[10] 7. A region of space is filled with a uniform electric field  $\vec{E} = 1.0 \text{ N/C } \hat{j} + 3.0 \text{ N/C } \hat{k}$ .  
 Consider a cube with edges of 2.0 m aligned with the x, y, and z axes as shown:



- [2] (a) What is the electric flux through the face, **A**, which is perpendicular to the x-axis?  
 [2] (b) What is the electric flux through the face, **B**, which is perpendicular to the y-axis?  
 [2] (c) What is the electric flux through the face, **C**, which is perpendicular to the z-axis?  
 [2] (d) What is the total electric flux through the surface of the cube?  
 [2] (e) What is the total charge enclosed by the cube surface?

$$\textcircled{a} \Phi_A = \int \vec{E} \cdot d\vec{A} = \int (1.0 \text{ N/C } \hat{j} + 3.0 \text{ N/C } \hat{k}) \cdot (dA \hat{i}) = 0 \frac{\text{N} \cdot \text{m}^2}{\text{C}}$$

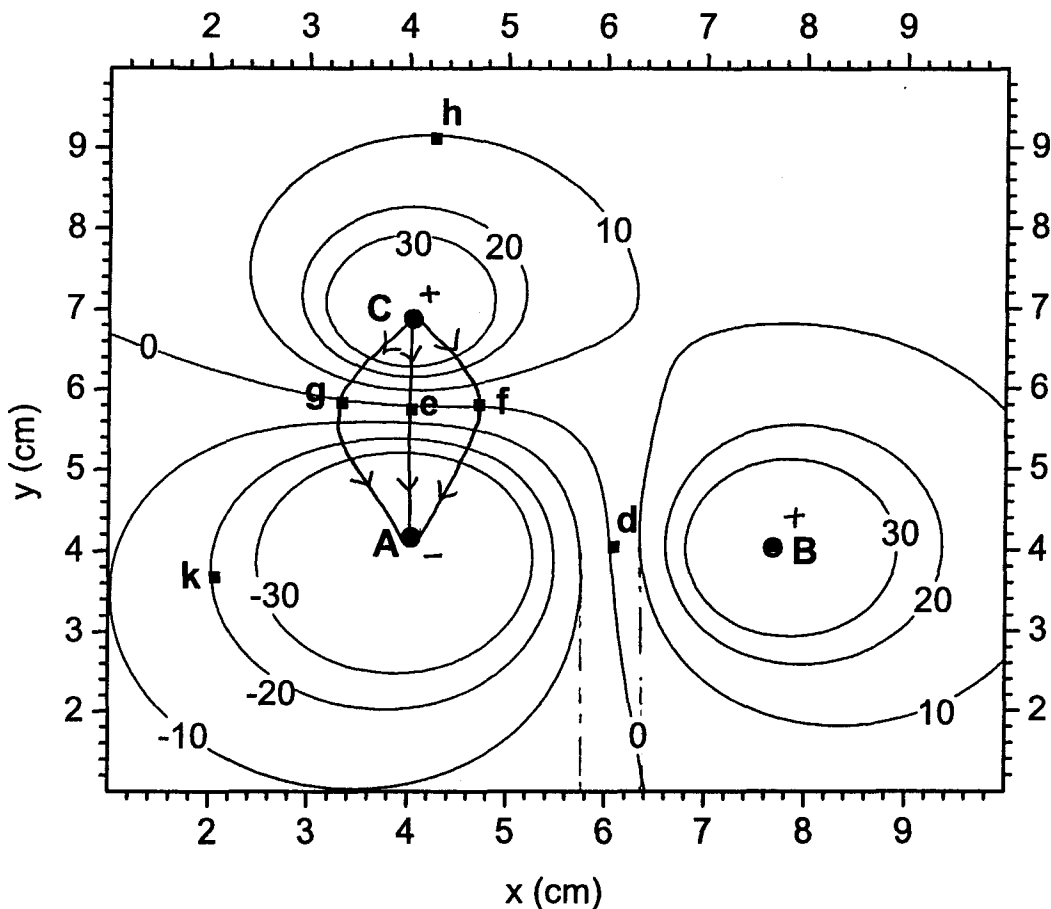
$$\textcircled{b} \Phi_B = \int \vec{E} \cdot d\vec{A} = \int (1.0 \text{ N/C } \hat{j} + 3.0 \text{ N/C } \hat{k}) \cdot (dA \hat{j}) = \int (1.0 \text{ N/C}) (dA) = (1.0 \text{ N/C}) (2\text{m})^2 = 4.00 \frac{\text{N} \cdot \text{m}^2}{\text{C}}$$

$$\textcircled{c} \Phi_C = \int \vec{E} \cdot d\vec{A} = \int (1.0 \frac{\text{N}}{\text{C}} \hat{j} + 3.0 \frac{\text{N}}{\text{C}} \hat{k}) \cdot (dA \hat{k}) = \int (3.0 \frac{\text{N}}{\text{C}}) dA = (3.0 \frac{\text{N}}{\text{C}}) (2\text{m})^2 = 12.0 \frac{\text{N} \cdot \text{m}^2}{\text{C}}$$

$$\textcircled{d} \Phi = \frac{q_{\text{INSIDE}}}{\epsilon_0} = \frac{0 \text{ C}}{\epsilon_0} = 0 \frac{\text{N} \cdot \text{m}^2}{\text{C}} \quad \text{or} \quad \Phi_{\text{TOTAL}} = \Phi_{+A} + \Phi_{-A} + \Phi_{+B} + \Phi_{-B} + \Phi_{+C} + \Phi_{-C} = (0 + 0 + 4 - 4 + 12 - 12) \frac{\text{N} \cdot \text{m}^2}{\text{C}} = 0 \frac{\text{N} \cdot \text{m}^2}{\text{C}}$$

$\textcircled{e}$  THE CUBE ENCLOSES NO CHARGED OBJECTS; TOTAL CHARGE ENCLOSED = 0.

- [10] 8. The graph below shows equipotential lines around three point charges; **A**, **B**, and **C**. The lines are drawn in steps of 10 V for all voltages between -30 V and +30 V. The potential is taken to be 0 at infinity.



- [2] (a) What are the signs of charge **A** and charge **B**? A IS NEGATIVE  
B IS POSITIVE
- [2] (b) Is the magnitude of the electric field larger at point **d** or at point **e**? Briefly explain your choice.
- [2] (c) Using the scale on the x-axis and the equipotentials, estimate the magnitude of the electric field at point **d**.
- [2] (d) Carefully draw the electric field lines that pass through points **e**, **f**, and **g**. Be sure to indicate direction and the charges on which the field lines start and end.
- [2] (e) How much work must an external agent do in order to move an electron from point **h** to point **k**?

(b)  $|\vec{E}| = \frac{\Delta V}{\Delta r}$ . At point **e**, the lines of constant potential are more closely spaced than they are at point **d**. Thus, at point **e** the change in potential with respect to distance is larger, and so is the magnitude of the electric field.

(c)  $|\vec{E}_d| = \frac{\Delta V}{\Delta x} = \frac{20V}{0.6cm} = 33.3 V/cm = 3330 V/m$

(e)  $W = -\Delta U = -q \Delta V = (-1.6 \times 10^{-19} C)(-20V - (+10V))$

$W = -(1.6 \times 10^{-19} C)(30V) = -4.80 \times 10^{-18} J$

$W_{\text{by ext. agent}} = +4.80 \times 10^{-18} J$

~~THIS IS~~ WORK BY ELECT FIELD  
 (must push electron to move from h to k)

[10] 9. A positive charge  $Q$  is uniformly distributed over a rod of length  $L$  situated along the  $x$ -axis. Point  $P$  is located a distance  $a$  from the left end of the rod along the  $x$ -axis as shown.

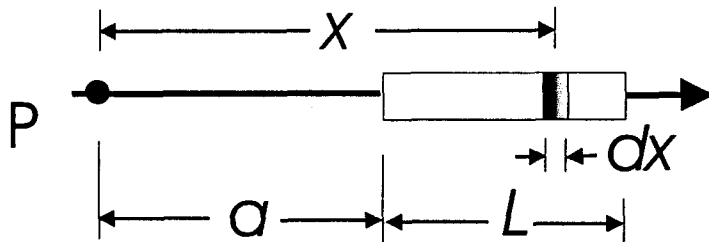
[6] (a) What is the contribution,  $dV$ , to the electric potential at  $P$  from an element of the rod of length  $dx$  located a distance  $x$  from point  $P$  as shown?

(b) Show that the electric potential at  $P$  due to the entire rod is given by

$$V = k_e \lambda \ln \left( 1 + \frac{L}{a} \right)$$

where  $\lambda = \frac{Q}{L}$ . (Hint: some integrals are provided on the cover page.)

[47] (c) Calculate the electric potential 35 cm along the  $x$ -direction from one end of the rod if the length of the rod is 57 cm and the total charge on the rod is  $5.5 \mu\text{C}$ .



(a)  $dV = k \frac{dq}{x} = k \frac{\frac{Q}{L} dx}{x} = k \lambda \frac{dx}{x}$ , where  $\lambda = \frac{Q}{L}$

(b)  $V = \lim_{N \rightarrow \infty} \sum_{i=1}^N dV = \int dV = \int_a^{a+L} k \lambda \frac{dx}{x} = k \lambda \int_a^{a+L} \frac{dx}{x}$

$$V = k \lambda \left[ \ln x \right]_{x=a}^{x=a+L} = k \lambda \left[ \ln(a+L) - \ln a \right] = k \lambda \ln \left( \frac{a+L}{a} \right) = k \lambda \ln \left( 1 + \frac{L}{a} \right)$$

(c)  $V = k \lambda \ln \left( 1 + \frac{L}{a} \right) = \left( 8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) \left( \frac{5.5 \times 10^{-6} \text{C}}{0.57 \text{m}} \right) \ln \left( 1 + \frac{0.57 \text{m}}{0.35 \text{m}} \right)$

$$V = 83,800 \text{ V}$$

**Some Potentially Useful Formulae and Constants:**

$$\vec{F}_{12} = k_e \frac{q_1 q_2}{r^2} \hat{r}_{12}$$

$$V = k_e \int \frac{dq}{r}$$

$$\vec{E} = k_e \frac{q}{r^2} \hat{r}$$

$$U_{12} = k_e \frac{q_1 q_2}{r_{12}}$$

$$\vec{E} = k_e \sum_i \frac{q_i}{r_i^2} \hat{r}_i$$

$$\Delta U = -q \int_A^B \vec{E} \cdot d\vec{s}$$

$$\vec{E} = k_e \int \frac{dq}{r^2} \hat{r}$$

$$\Delta V = V_B - V_A = - \int_A^B \vec{E} \cdot d\vec{s}$$

$$\Phi_E = \int \vec{E} \cdot d\vec{A}$$

$$\Delta U = q\Delta V$$

$$\Phi_E = \frac{q_{\text{inside}}}{\epsilon_0}$$

$$\vec{E} = - \left( \frac{dV}{dx} \hat{i} + \frac{dV}{dy} \hat{j} + \frac{dV}{dz} \hat{k} \right)$$

$$V = k_e \frac{q}{r}$$

$$R = \frac{\Delta V}{I}$$

$$V = k_e \sum_i \frac{q_i}{r_i}$$

$$R = \rho \frac{l}{A}$$

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

**Physical constants:**

$$k_e = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2$$

$$e = 1.602 \times 10^{-19} \text{ C}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 / \text{N} \cdot \text{m}^2$$

$$m_e = 9.11 \times 10^{-31} \text{ kg}$$

**Mathematical formulae:**

$$\int \frac{dr}{r^2} = -\frac{1}{r}$$

$$\int \frac{dx}{(x^2 + y^2)^{3/2}} = \frac{x}{y^2 \sqrt{x^2 + y^2}}$$

$$\int \frac{dx}{x} = \ln x$$

$$\int \frac{x dx}{(x^2 + y^2)^{3/2}} = -\frac{1}{\sqrt{x^2 + y^2}}$$

$$\int \frac{dx}{\sqrt{x^2 + y^2}} = \ln \left[ x + \sqrt{x^2 + y^2} \right]$$

$$\vec{A} \cdot \vec{B} = AB \cos \theta = A_x B_x + A_y B_y + A_z B_z$$

$$\vec{A} \times \vec{B} = (A_y B_z - A_z B_y) \hat{i} + (A_z B_x - A_x B_z) \hat{j} + (A_x B_y - A_y B_x) \hat{k}$$