Electric Force and Potential Energy

For a charge q_0 in an electric field:

The force picture

$$\vec{F} = q_0 \vec{E}$$

Can we similarly look for an energy picture ?

The energy picture (the work-kinetic energy theorem)

$$W_{net} = q_0 \int_i^f \vec{E} . d\vec{s} \quad \text{and} \quad K_f - K_i = W_{net}$$

$$\Delta U = U_f - U_i = -W_{net} = -q_0 \int_i^f \vec{E} . ds$$

Example: Projectile Motion

We will do a familiar problem again – projectile motion in the presence of a gravitational force – in two ways.

The force picture

$$\vec{F} = m\vec{g} = -mg\hat{k}$$

The energy picture (the work-kinetic energy theorem)

$$W_{net} = \int_i^f \vec{F} . d\vec{s}$$

is work done by the field on the mass. Positive work reduces the potential energy.

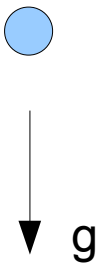
$$\Delta U = U_f - U_i = -W_{net}$$

Positive work increases the kinetic energy.

$$K_f - K_i = W_{net}$$

Gravitational Force and Potential Energy

For a mass *m* in an gravitational field:



The force picture

$$\vec{F} = m\vec{g} = -mg\hat{k}$$

Can we similarly look for an energy picture ?

The energy picture (the work-kinetic energy theorem)

$$W_{net} = -mg\int_i^f dz = -mg(z_f - z_i)$$

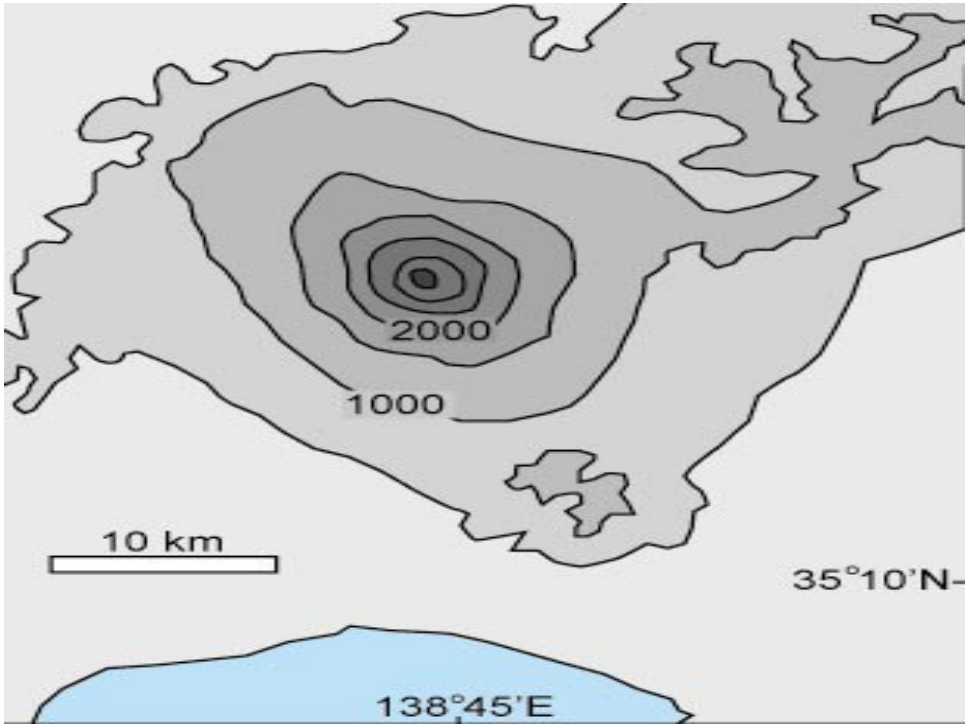
$$\Delta U = U_f - U_i = -W_{net} = mg(z_f - z_i)$$

$$\Delta U = mg\Delta z$$

Contour Maps

are lines of ***constant height***
and ***constant gravitational potential energy****.*

Mt. Fuji Photo, Japan. [Wikipedia]



Mt. Fuji Contour Map, Japan. [Wikipedia]

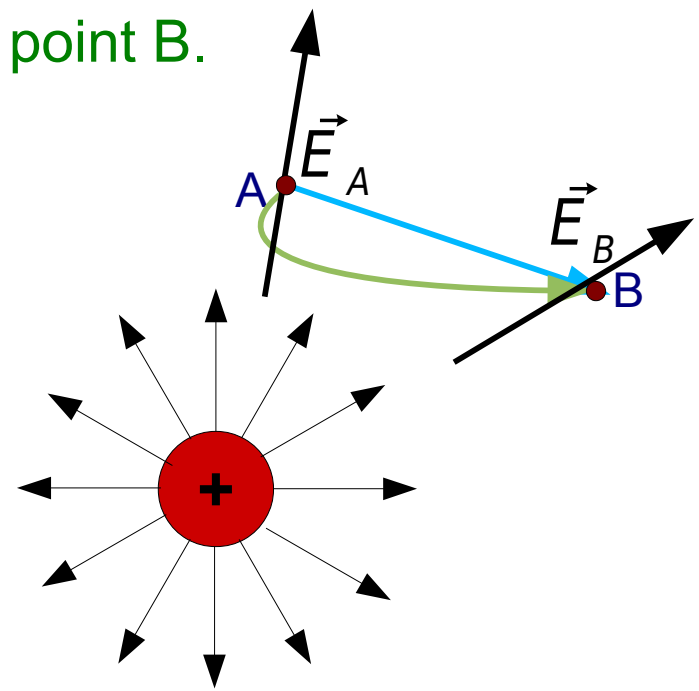
Electric Potential Difference

Lets say we move a test charge from point A to point B.

$$\Delta V = -\int_A^B \vec{E} \cdot d\vec{s}$$

For a point charge:

$$\vec{E} = k_e \frac{q}{r^2}$$



$$\Delta V = -\int_A^B \vec{E} \cdot ds = -k_e q \int_A^B \frac{1}{r^2} \hat{r} \cdot d\vec{s}$$

The path denoted by the vector $d\vec{s}$ does not have to have to be along the field. It is any path between the two paths A and B. It does not matter which path you choose.

In general the above integral can be complicated.

Units

$$\vec{F} = q\vec{E}$$

Force has units of N (newtons)
Charge has units of C (Coulombs)
So electric field is N/C

$$\Delta V = -\int_A^B \vec{E} \cdot d\vec{s}$$

Potential difference is
(electric field) x (distance)
(N/C).m which is J/C (joules per coulomb)
This is also denoted as V (volts)

Therefore, units of electric field can also be written as V/m.

Electric field: both N/C and V/m is correct.

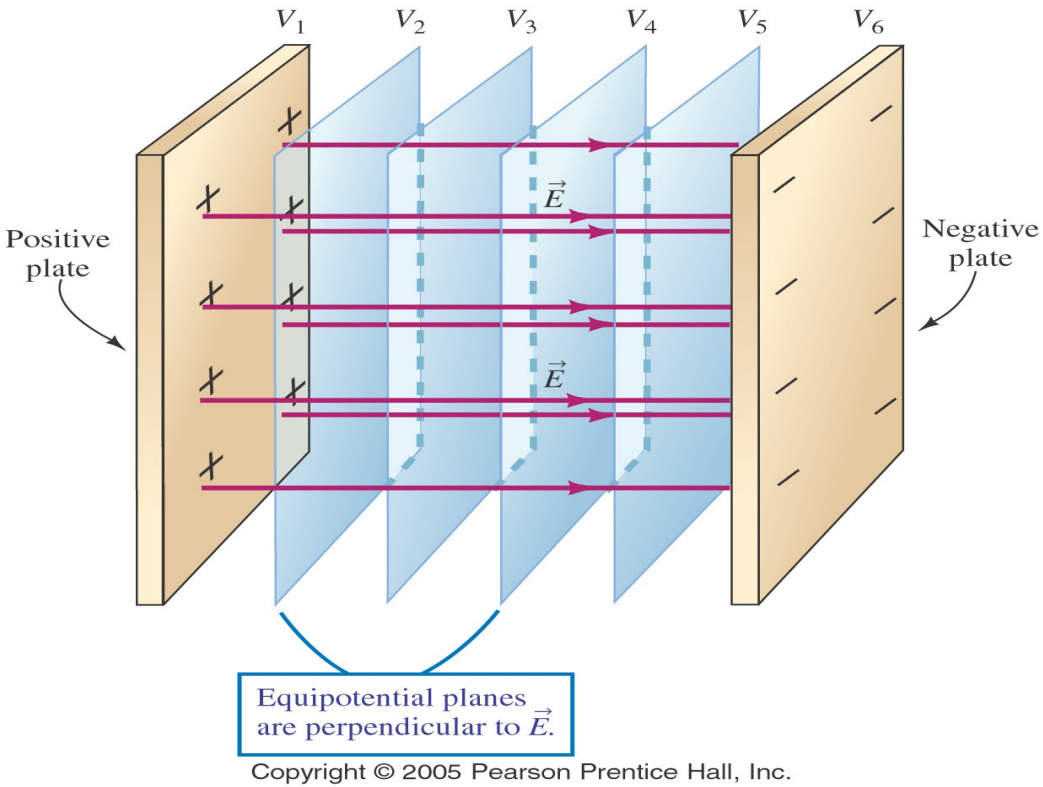
Electric potential: both J/C and V is correct.

Electric Potential Energy

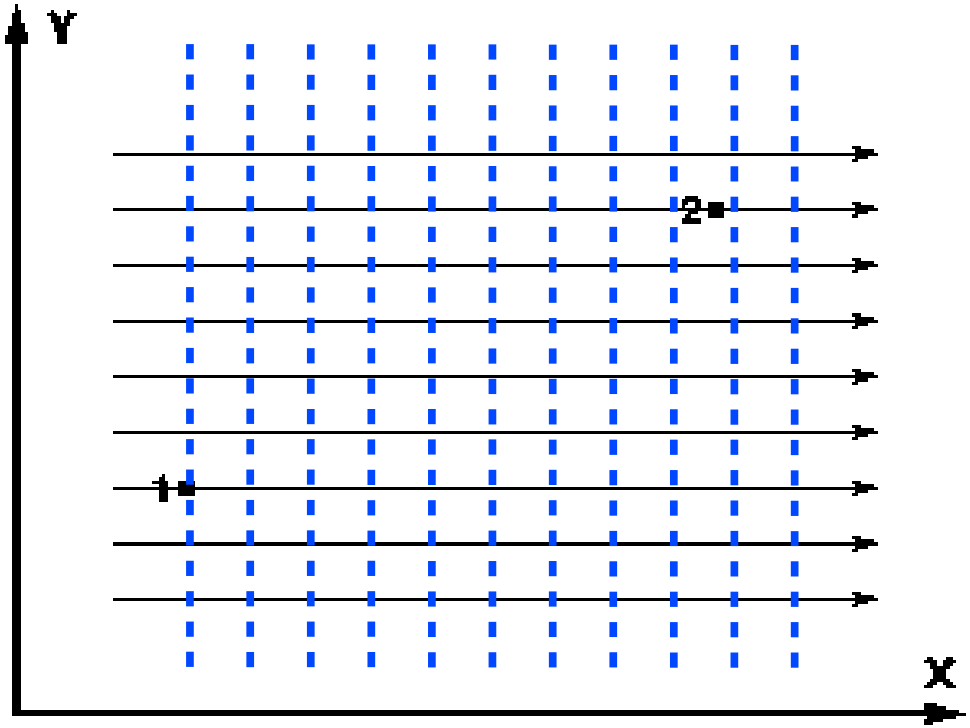
The change in potential *energy* is proportional to the change in electric potential

$$\Delta U_E = q \Delta V$$

Equipotentials are Voltage Contours

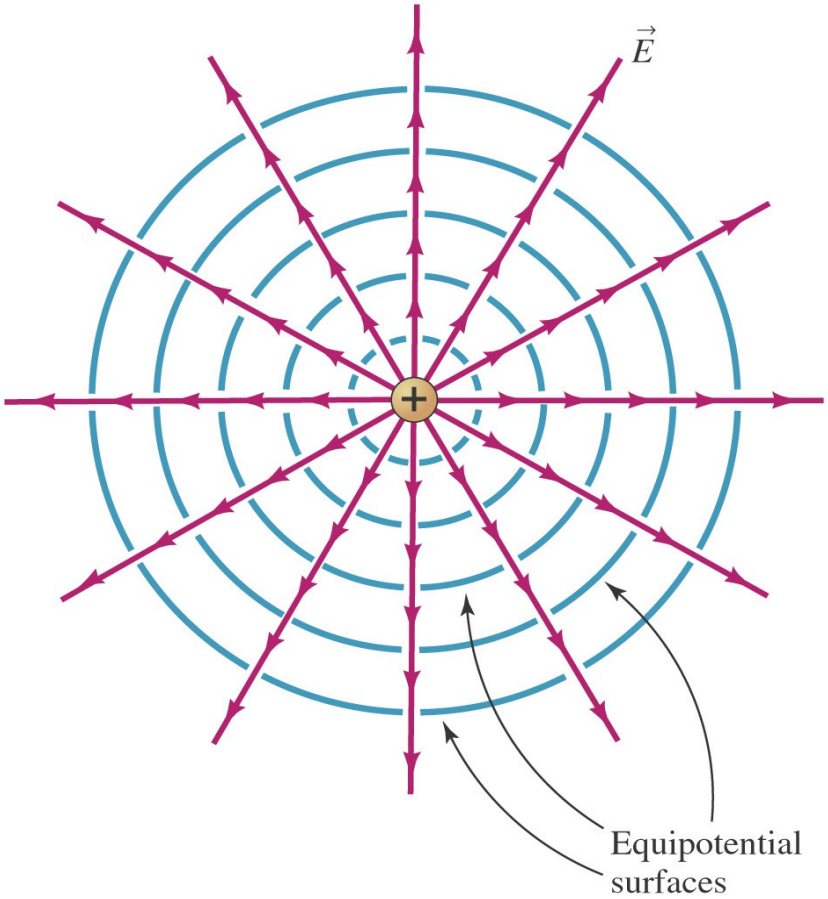


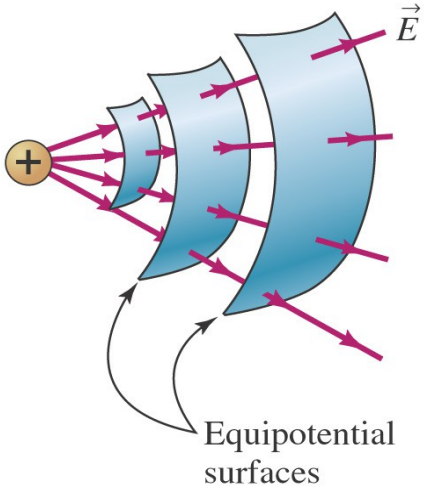
Uniform field between Two Charged Parallel Plates



Constant Electric Field and Equipotential Lines

Equipotentials are Voltage Contours





The Electric Field is the Electric Potential Gradient

$$\Delta V = -\int_A^B \vec{E} \cdot d\vec{s}$$

Given the electric field: we can figure out the potential difference.

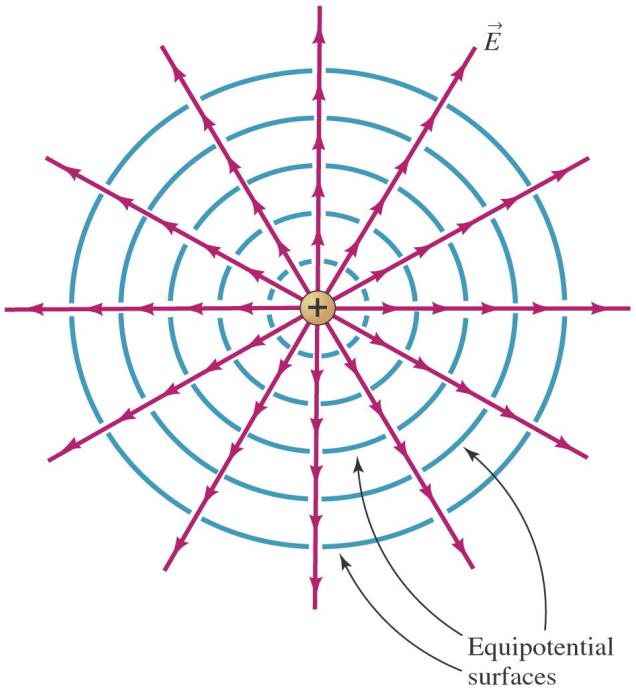
Now the inverse problem:
Given the electric potential: how do we figure out the electric field?

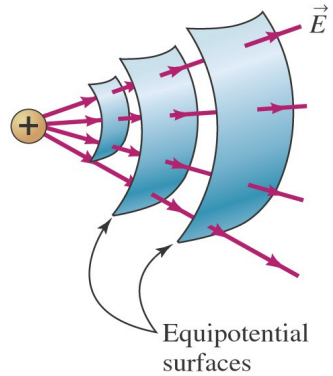
$$E = \frac{k_e q}{r^2}$$

$$V = k_e \frac{q}{r}$$

$$\vec{E}(r) = \frac{-\partial V}{\partial r}\hat{r}$$

The Electric Field is the Electric Potential Gradient





$$\vec{E}(r) = \frac{-\partial V}{\partial r}\hat{r}$$

Why the partial derivative ?

$$\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$$
$$= \vec{E}(x, y, z)$$

in Cartesian coordinates, or

$$\vec{E} = \vec{E}(r, \theta, \phi)$$

in spherical coordinates

In general: $\vec{E}(x, y, z) = -\vec{\nabla} V = -\left[\hat{i}\frac{\partial V}{\partial x} + \hat{j}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z}\right]$

Electric Potential on the Surface of a Charged Conductor

Zero Electric Field: Inside an electrical conductor, in electrostatic equilibrium, there is no net electric field. All the charge resides on the surface.

The field lines point normal to the surface at every point on the surface.

Equipotential: On the surface of an electrical conductor, in electrostatic equilibrium, there is a constant electrical potential.

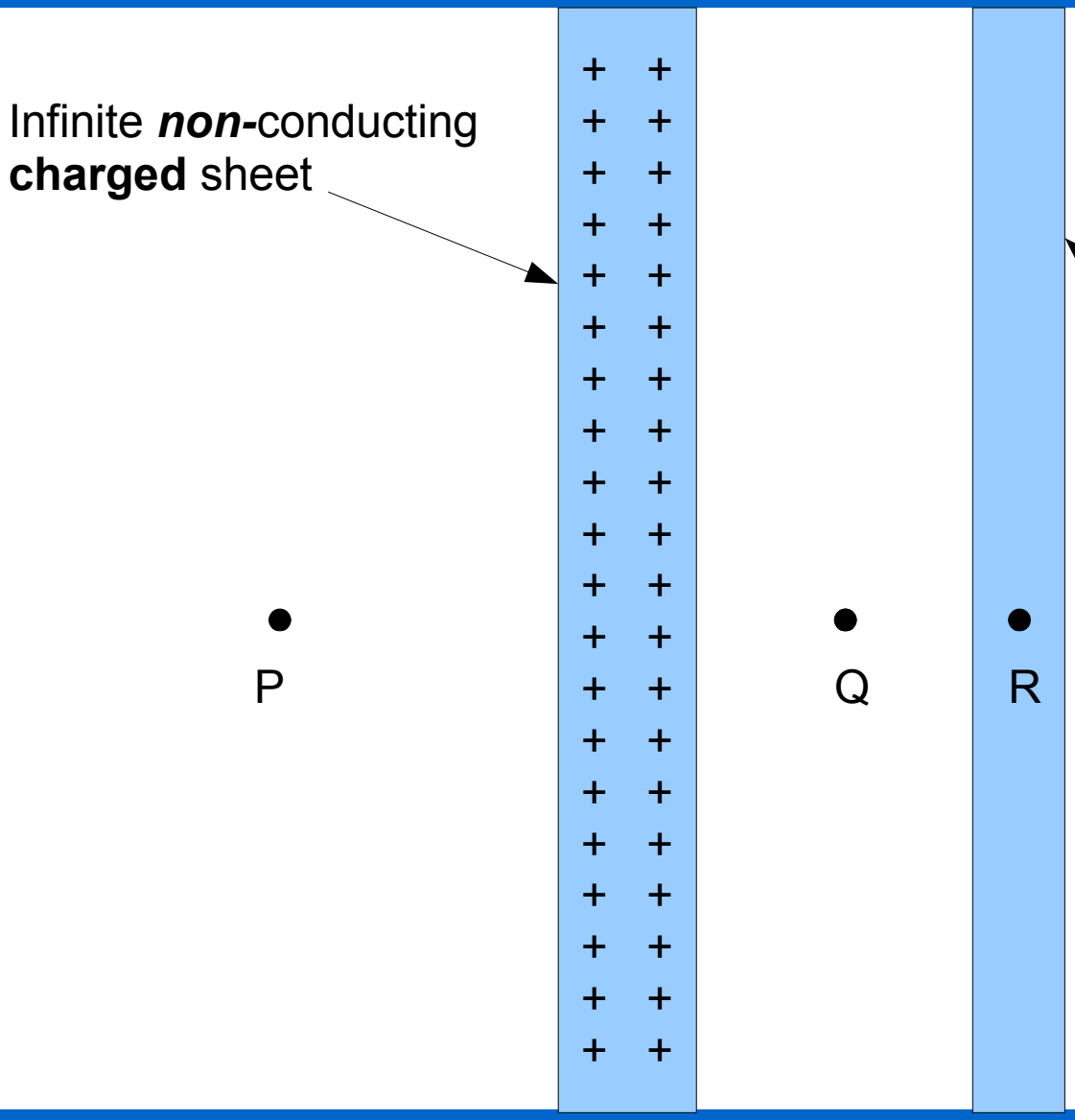
$$\Delta V = -\int_A^B \vec{E} \cdot d\vec{s}$$

$d\vec{s}$ is tangential (parallel) to the surface

$\vec{E}$ is perpendicular to the surface

$$\Delta V = -\int_A^B \vec{E} \cdot d\vec{s} = 0$$

Conductors and Gauss' law



Zero Electric Field: Inside an electrical conductor, in electrostatic equilibrium, there is no net electric field. All the charge resides on the surface.

Infinite ***conducting uncharged*** sheet

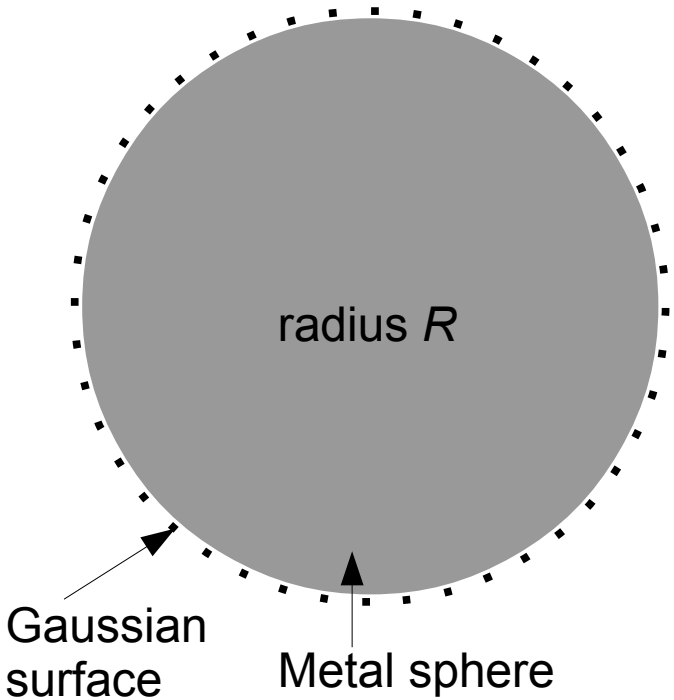
What is the field at points P, Q, R and S?

The van de Graaff Electrostatic Generator

Credit: Edward Hayden, Feb 17, 2010.

A willing student holds on to the metal sphere. The charge makes his hair repel and stand up.

Conductors: Electric Potential and Charge



A metal sphere (radius r_1) has an electrical potential that is equal everywhere on its surface

How to relate electric field, electric potential and electric charge?

1. Closed surface : begin with Gauss' Law
Imagine drawing a Gaussian surface just outside the sphere
2. From Gauss' Law the flux is:

$$\phi_E = \frac{Q}{\epsilon_0} = E\left(4\pi R^2\right)$$

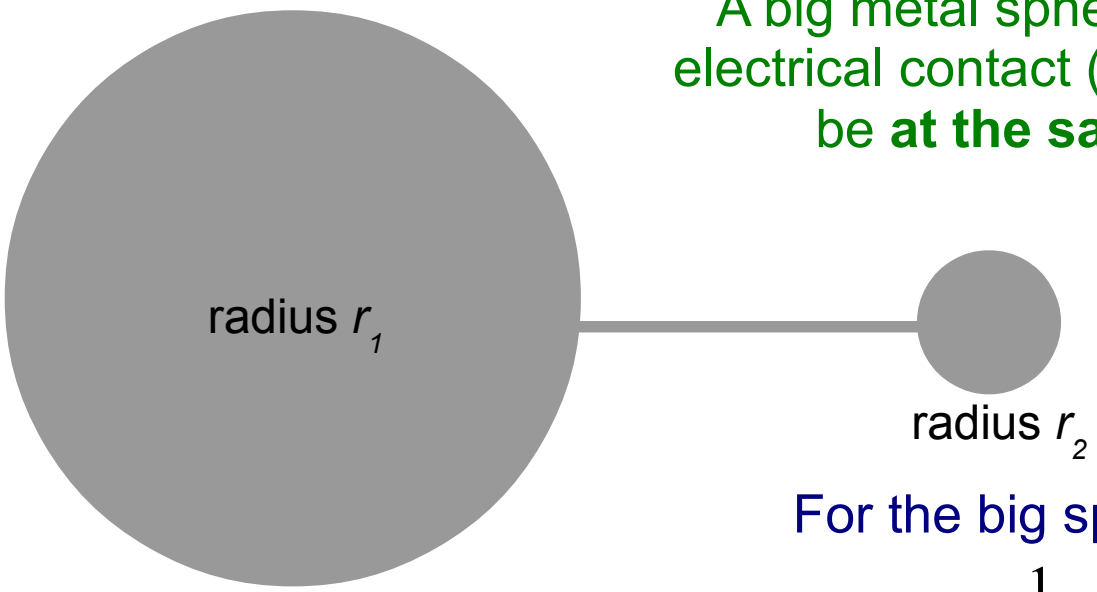
3. The electric field is: $E = \frac{1}{4\pi\epsilon_0}\frac{Q}{R^2}$

4. The electric potential is: $V = \int_{\infty}^{R} \vec{E}\,.\,d\vec{s} = \frac{1}{4\pi\epsilon_0}\frac{Q}{R}$

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{R}$$

Conductors: Electric Potential and Charge

A big metal sphere and a small metal sphere in electrical contact (e.g. with a wire joining them) will be **at the same electrical potential *V*.**



For the big sphere:

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q_1}{r_1}$$

For the small sphere:

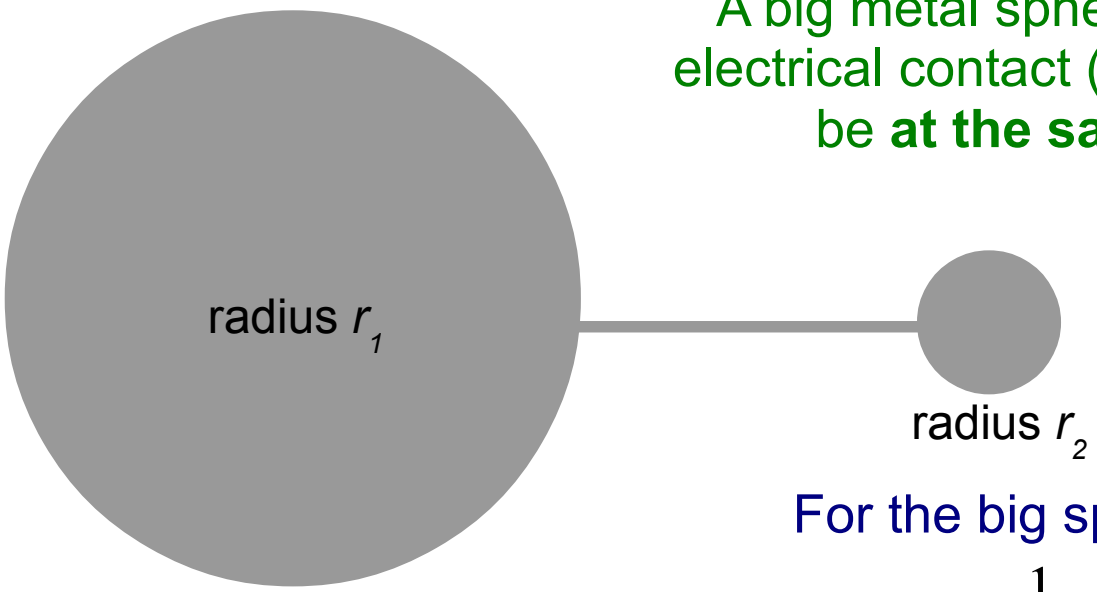
$$V = \frac{1}{4\pi\epsilon_0}\frac{Q_2}{r_2}$$

The big sphere has more charge than the small one.

$$\boxed{\frac{Q_1}{r_1} = \frac{Q_2}{r_2}}$$

Conductors: Electric Potential and Electric Field

A big metal sphere and a small metal sphere in electrical contact (e.g. with a wire joining them) will be **at the same electrical potential *V*.**



For the big sphere:

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q_1}{r_1}$$

$$E_1 = \frac{1}{4\pi\epsilon_0}\frac{Q_1}{r_1^{\,2}} = \frac{V}{r_1}$$

For the small sphere:

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q_2}{r_2}$$

$$E_2 = \frac{V}{r_2}$$

The field on the big sphere is smaller! The smaller the radius of curvature the higher the field. Hence, lightning rods are made sharp.

$$\boxed{E_1 r_1 = E_2 r_2}$$

Conductors:
Electric Potential and Electric Field

Earth ground

Negative
Electric
potential

Why the spark ?

Air is normally a very good electric insulator. But when the electric field is high, this induces breakdown (called “dielectric breakdown”) of the air. The “fresh air” smell during a thunderstorm comes from ozone after dielectric breakdown.

The left sphere is the “lightning rod” :
the smaller it is, the easier it is to generate sparks.