ELECTRIC POTENTIAL (Chapter 20)

In mechanics, saw relationship between conservative force and potential energy:

\[ F_x = -\frac{dU}{dx} \]

**Says:** How **scalar quantity** (potential energy) depends on position gives components of a **vector quantity** (force)

Any connection to electric forces? Consider an arrangement of charges:

- Have learned to **directly** calculate electric field at a point by adding (vector sum) contributions (vectors) to field from each charge

- Will now learn to calculate **electric potential** (potential energy per unit charge) at a point by a **scalar sum** and then get **components** of electric field from **derivatives**
  - Avoids vector sum. Scalar sums are easier!

**Electric Potential is like landscape.** \( \vec{E} \) is a vector pointing down (or up) hill.
**1st step:** Change in **Potential Energy** of a charge moving in an electric field.

<table>
<thead>
<tr>
<th>Electrostatic force on charge $q_0$ in field $\vec{E}$ is $\vec{F} = q_0 \vec{E}$</th>
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<tbody>
<tr>
<td>If charge displacement is $d\vec{s}$, then work <strong>ON</strong> charge <strong>BY</strong> electric field is:</td>
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<tr>
<td>$dW = \vec{F} \cdot d\vec{s} = q_0 \vec{E} \cdot d\vec{s}$</td>
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<td>• Notice scalar (dot) product</td>
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<td><strong>But</strong> electrostatic force is <strong>conservative:</strong></td>
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<td>• <strong>Potential energy</strong> ($U$) decreases when conservative force does <strong>positive work</strong></td>
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<td><strong>So:</strong> $dW = -dU = q_0 \vec{E} \cdot d\vec{s}$</td>
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<td>• $dU$ is the <strong>change</strong> in potential energy of charge when it is displaced by $d\vec{s}$</td>
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<td>If charge is moved from point A to point B, <strong>change</strong> in potential energy is:</td>
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<tr>
<td>$\Delta U = U_B - U_A = -q_0 \int_A^B \vec{E} \cdot d\vec{s}$</td>
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<tr>
<td>• <strong>Important:</strong> Force is conservative <strong>SO</strong> $U$ is independent of path from $A \rightarrow B$</td>
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So: $U_A$ is the electric potential energy of $q_0$ at A. Depends on two things:

- Charges surrounding point A (a property of the space around A)
- The charge $q_0$ (property of the charge at A)

2\textsuperscript{nd} step: Define Electric Potential $V_A$:

- Electric Potential at some point A is Electric potential energy per unit charge at point A

\[ V_A = \frac{U_A}{q_0} \]

\begin{itemize}
  \item $U_A$ is potential energy of $q_0$ at point A
  \item $V_A$ is a property of location A due to surrounding charges (but not $q_0$)
\end{itemize}
ELECTRIC POTENTIAL DIFFERENCE:

**Difference** in electric potential **between** points A and B is:

\[ \Delta V = V_B - V_A = \frac{\Delta U}{q_0} \]

**SO:**

\[ \Delta V = V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{s} \]

- Potential difference between two points depends on electric field
- Note sign and order of integration limits

Can now calculate Electric Potential **Difference**:

- To find Electric Potential, **MUST** specify location (reference) where \( V = 0 \)
  - Will also be location where \( U = 0 \)
  - Choice of reference depends on situation. Need to indicate clearly
TWO CONVENTIONS FOR CHOOSING REFERENCE (where $V = 0$)

- For a POINT IN SPACE, can choose $V = 0$ to be at infinity
  
  o So electric potential at point P is

  $$V_p = -\int_{\infty}^{P} \vec{E} \cdot d\vec{s}$$

  o This is the work to bring a unit charge test particle from infinity to P

- In an electric circuit can choose a specific circuit point to be $V = 0$
  
  o Often one terminal of a battery or a point connected to ground.
UNIT FOR ELECTRIC POTENTIAL: VOLT

- Define: 1V = 1J/C
- To move 1C from \( V = 1V \) to \( V = 0V \) (i.e. through \( \Delta V = -1V \)) takes 1J of work

- Gives alternate unit for electric field \( \vec{E} \)
  - Can express electric field in V/m. 1 N/C = 1 V/m

- Also gives alternate unit for energy: electron-volt
  - Define 1 eV as kinetic energy gained by an electron accelerated by an electric field through a 1V potential difference
  - Charge of one electron is \(-1.6 \times 10^{-19} \) C so:
    - 1 eV = \( 1.6 \times 10^{-19} \) C x 1 J/C = \( 1.6 \times 10^{-19} \) J
ELECTRIC POTENTIAL DIFFERENCE IN A UNIFORM ELECTRIC FIELD

Consider points A, B and B’ in a uniform $\vec{E}$ field:

- A and B separated by displacement $\vec{d}$ parallel to $\vec{E}$
- B and B’ are placed on a line perpendicular to $\vec{E}$
- Compare $V_B - V_A$ to $V_{B'} - V_A$

\[
V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{s}_{AB}
= -\vec{E} \cdot \int_A^B ds_{AB}
= -\vec{E} \cdot \Delta \vec{r}_{AB}
= -Ed
\]

\[
V_{B'} - V_A = -\int_A^{B'} \vec{E} \cdot d\vec{s}_{AB'}
= -\vec{E} \cdot \int_A^{B'} ds_{AB'}
= -\vec{E} \cdot \Delta \vec{r}_{AB'}
= -E |\Delta \vec{r}_{AB'}| \cos \theta
= -Ed
\]
So: \[ V_B - V_A = V_{B'} - V_A \] which says that \( V_B = V_{B'} \)

Two things to notice:
- **1\textsuperscript{st}: EQUIPOTENTIAL**
  - All points on plane perpendicular to uniform \( \vec{E} \) field have **same electric potential**
    - Plane perpendicular to uniform \( \vec{E} \) is called an **equipotential plane**
    - It requires **no work** to move a charge along an equipotential plane
    - will talk about equipotential for non-uniform \( \vec{E} \) later.
• 2\textsuperscript{nd}: SIGN OF $V$
  
  o $\Delta V = V_B - V_A = -Ed$
    
    ▪ Says that if $A \rightarrow B$ is in same direction as $\vec{E}$, then $\Delta V = V_B - V_A < 0$

  o change in potential energy of $q_0$ when moved from $A \rightarrow B$ is

  $\Delta U = q_0\Delta V = -q_0Ed$

Behaviours of positive and negative charges in uniform electric field

  o If path is in the same direction as $\vec{E}$,
    
    ▪ then potential difference $\Delta V$ is negative

    ▪ change in electric potential energy $\Delta U$ depends on sign of charge
**Positive charge in a uniform field**

- For a positive charge moving in direction of \( \vec{E} \)
  - i.e. dir. for which \( \Delta V \) is negative

\[
\Delta U = q_0 \Delta V < 0
\]

- Says that **electric potential energy** of a **positive charge** decreases when it moves in direction of \( \vec{E} \) (charge accelerates)

- Check: positive charge released from rest in uniform \( \vec{E} \)
  - Accelerates in direction of \( \vec{E} \)
  - Kinetic energy \( K \) increases
  - Potential energy \( U \) decreases
  - Mechanical energy \( E = K + U \) stays constant
    - if only force acting is electric
Negative charge in a uniform field

- For a negative charge moving in direction of $\vec{E}$
  - i.e. dir. for which $\Delta V$ is negative

\[ \Delta U = q_0 \Delta V > 0 \]

- Says that electric potential energy of a negative charge increases when it moves in direction of $\vec{E}$ (needs to be pushed)

- Check: negative charge released from rest in uniform $\vec{E}$
  - Accelerates in direction opposite to $\vec{E}$
    - i.e. direction for which $\Delta U = q_0 \Delta V < 0$
      - for a negative charge
  - Mechanical energy $E=K+U$ stays constant
    - if only force acting is electric
Behaviours of positive and negative charges in uniform electric field