MAGNETIC FIELD AROUND CURRENT-CARRYING WIRES

How will we tackle this? Plan:

1\textsuperscript{st}: Will look at contribution $d\vec{B}$ to the total magnetic field at some point in space due to the current in a small segment $d\vec{s}$ of wire

- Biot-Savart Law

2\textsuperscript{nd}: Will use Biot-Savart Law to find $\vec{B}$ along the axis of a current loop.

3\textsuperscript{rd}: Will use Biot-Savart Law to get result for field around a “long” wire

- $B = \frac{\mu_0 I}{2\pi a}$ dir. by RH-rule, $\mu_0 = 4\pi \times 10^{-7}$ T $\cdot$ m/A

4\textsuperscript{th}: Will look at force between parallel current-carrying wires

5\textsuperscript{th}: Ampere’s Law: another way to get $B = \frac{\mu_0 I}{2\pi a}$ for field around long wire
BIOT-SAVART LAW (22.7)

Expression for contribution $d\vec{B}$ to magnetic field at $P$ due to current $I$ in small segment $d\vec{s}$ of wire

- $\hat{r}$ is UNIT VECTOR pointing $d\vec{s} \rightarrow P$
- $r$ is DISTANCE from $d\vec{s} \rightarrow P$
- $\theta$ is the ANGLE between $d\vec{s}$ and $\hat{r}$

Then contribution to $\vec{B}$ at $P$ from $d\vec{s}$ is

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I \; ds \times \hat{r}}{r^2}$$

- $\mu_0 = 4\pi \times 10^{-7}$ T $\cdot$ m/A is the permittivity of free space
- Direction of $d\vec{B}$ is from $d\vec{s} \times \hat{r}$ (can also use RH-rule with thumb $\rightarrow$ current)
**BIOT-SAVART LAW:** understanding

\[ d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\,d\vec{s} \times \hat{r}}{r^2} \]

- For drawing, direction of \( d\vec{s} \times \hat{r} \) is out of screen/page
  - Fingers sweep \( d\vec{s} \rightarrow \hat{r} \); thumb shows direction of \( d\vec{s} \times \hat{r} \)
- So \( d\vec{B} \) at \( P \) due to \( d\vec{s} \) points out of screen/page
- Magnitude \( |d\vec{s} \times \hat{r}| = ds \sin \theta \)
  - For a given \( r \), contributions \( d\vec{B} \) from \( d\vec{s} \) are maximum for points on plane perpendicular to \( d\vec{s} \)
  - Current in \( d\vec{s} \) makes **NO** contribution to \( d\vec{B} \) at points along direction \( d\vec{s} \)
- For **TOTAL FIELD** at \( P \), must sum (integrate) contributions from all segments of wire (don’t panic. We will only do special cases)
EXAMPLE: 22.6 in text – Magnetic field on axis of circular current loop

Look at a loop of radius \( r \) located in the \( yz \) plane carrying a current \( I \). What is the magnetic field a distance \( x \) from the centre of the loop along its axis?
MAGNETIC FIELD AROUND A LONG (INFINITE) WIRE

Biot-Savart Law gives contribution \( dB \) at \( P \) due to current \( I \) in segment \( ds \)

\[
dB = \frac{\mu_0}{4\pi} \frac{I ds \times \hat{r}}{r^2}
\]

- \( dB \) points out of screen/page at \( P \) by RH-rule

To get \( \vec{B} \) at \( P \) due to WHOLE wire, must sum contributions from all angles

- Angle between \( ds \) and \( \hat{r} \) changes with position along wire!
  - Will state result and then sketch derivation
MAGNETIC FIELD AROUND A LONG (INFINITE) WIRE

- Result of using Biot-Savart Law:
  - Magnetic field lines circle wire → no component of $\vec{B}$ parallel to wire

- Magnitude of $\vec{B}$ inversely proportional to perpendicular $a$ distance from wire
  \[ |\vec{B}| = \frac{\mu_0 I}{2\pi a} \]  
  (IMPORTANT RESULT)

- Direction of magnetic field lines:
  - Another Right-Hand Rule: thumb along $I$; fingers curl in direction of $\vec{B}$
“Sketch” of how we get $|\vec{B}| = \frac{\mu_0 I}{2\pi a}$ from the Biot-Savart Law

Looking for $\vec{B}$ at a point $P$ located perpendicular distance $a$ from wire

- Start with contribution to field at $P$ due to segment $d\vec{s}$ located distance $s$ along the wire from the point closest to $P$:

  $$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I \, d\vec{s} \times \hat{r}}{r^2}$$

  - Note that $d\vec{s} \times \hat{r}$ points out of the screen/page and that $|d\vec{s} \times \hat{r}| = ds \sin \theta$

To get total $\vec{B}$ at $P$ from infinite wire, could sum $d\vec{B}$ from $s = -\infty$ to $s = +\infty$.

- Helps to express $r$ and $ds$ in terms of angle $\theta$ between $d\vec{s}$ and $\hat{r}$ and then sum from $\theta = 0$ to $\theta = \pi$
“Sketch” of how we get \( \vec{B} = \frac{\mu_0 I}{2\pi a} \) from the Biot-Savart Law (continued)

Changing Variables:

- From \( \sin \theta = \frac{a}{r} \) get \( r = \frac{a}{\sin \theta} \)

- Tricky point: for \( \theta \) as shown, \( s \) is negative so that \( \tan \theta = -\frac{a}{s} \) or \( s = -a \cot \theta \)

- Use derivative to relate \( ds \) to \( d\theta \)

  - From \( \frac{ds}{d\theta} = a \csc^2 \theta = \frac{a}{\sin^2 \theta} \) we get \( ds = \frac{a d\theta}{\sin^2 \theta} \)
“Sketch” of how we get \( \left| \vec{B} \right| = \frac{\mu_0 I}{2\pi a} \) from the Biot-Savart Law (continued)

**Doing the integral.** So far, we have:

- \( r = \frac{a}{\sin \theta} \)

- \( ds = \frac{ad\theta}{\sin^2 \theta} \) so \( |d\vec{s} \times \hat{r}| = ds \sin \theta = \frac{a \, d\theta}{\sin \theta} \)

So contribution to field at \( P \) from \( d\vec{s} \) is

\[
dB = \frac{\mu_0 I}{4\pi} \frac{d\vec{s} \times \hat{r}}{r^2} = \frac{\mu_0 I}{4\pi a} \sin \theta \, d\theta
\]

Magnitude of total field is:

\[
B = \frac{\mu_0 I}{4\pi a} \int_0^\pi \sin \theta \, d\theta = \left[ -\frac{\mu_0 I}{4\pi a} \cos \theta \right]_0^\pi = \frac{\mu_0 I}{2\pi a}
\]

**RESULT:** field at distance \( a \) from wire with current \( I \):

\[
B = \frac{\mu_0 I}{2\pi a}
\]
USE \[ B = \frac{\mu_0 I}{2\pi a} \] TO FIND MAGNETIC FORCE BETWEEN PARALLEL WIRES

• Field at \( I_1 \) due to \( I_2 \) is \( B_2 = \frac{\mu_0 I_2}{2\pi a} \)
  
  o Exerts a force on length \( l \) of \( I_1 \): \( \vec{F}_1 = I_1 \vec{l} \times \vec{B}_2 \)

  o Magnitude of force on length \( l \) of \( I_1 \) is \( F_1 = I_1 l B_2 = \frac{\mu_0 l I_1 I_2}{2\pi a} \)
MAGNETIC FORCE BETWEEN PARALLEL WIRES

- Force on $I_1$ per unit length is $\frac{F_1}{l} = \frac{\mu_0 I_1 I_2}{2\pi a}$ toward $I_2$ (for currents in same dir.)

By Newton's 3rd law,

- Force on $I_2$ per unit length is $\frac{F_2}{l} = \frac{\mu_0 I_1 I_2}{2\pi a}$ toward $I_1$ (for currents in same dir.)

In general:
- For parallel conductors, **current in same direction**: 
  - Wires **ATTRACT** with force per unit length $\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi a}$

- For parallel conductors, **current in opposite direction**: 
  - Wires **REPEL** with force per unit length $\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi a}$

Provides definition of **AMPERE**:
- For 2 wires, 1 m apart, $I_1 = I_2 = 1\, \text{A}$ → force per unit length is $2 \times 10^{-7} \, \text{N/m}$