

**Investigating vortex streets behind real and virtual bluff bodies: An experiment for
an advanced undergraduate laboratory**

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Abstract

This paper describes a laboratory experiment designed to study regular arrays of vortices occurring behind an object in a stream of fluid. This phenomenon is observed in industrial flows, flows in the ocean and in the atmosphere. We consider the flow behind a circular cylinder. In the second part of the experiment the effect of the body on the fluid is imitated by using an appropriate force field when there is no real body present in the fluid. The force field (virtual body) is created by a permanent magnet located above the surface of water in combination with electric current applied in the horizontal direction. The apparatus required for the experiment is inexpensive and easy to construct.

I. INTRODUCTION.

The phenomenon of regular vortex shedding behind bodies in a stream of fluid is often observed in nature. For example, vortex streets can be observed on a large scale in satellite images of cloud cover behind mountainous islands. On a smaller scale, the flutter of a flag in the wind or the generation of sound by a wire are all related phenomena.

We consider the classical fluid mechanics problem of flow behind a circular cylinder. This problem has been a subject of serious investigation for over hundred years¹. The underlying (nonlinear) phenomena which cause instability in the form of the vortex shedding are quite complex. However, the vortex street itself is simple, highly periodic and robust. The characteristics of these flows can be easily measured and analyzed using dimensional analysis. A body in a stream experiences a drag force. The effect of the body on the fluid is therefore described by the force equal to the drag force in magnitude and acting in the opposite direction. We can create a “virtual” body by imitating this force when there is no real body in the fluid. An electromagnetic method is used for this purpose.

The goal of this paper is to describe a simple, low-cost experiment for the advanced undergraduate laboratory and to introduce students to methods of dimensional analysis. While the experiment is simple enough to be performed by undergraduates, it exposes students to the kinds of methods that are often encountered in advanced research.

II. EXPERIMENT

The apparatus is illustrated schematically in Fig. 1. The flows are generated in a Plexiglas tank, 50 cm long, 10 cm deep and 20 cm wide. The tank contains two layers of salt water, each of depth 0.5 cm and of different salt concentration, the lower layer being more dense than the upper one. Two layers are used to minimize the vertical component of velocity, constraining the flow to two dimensions². Two copper pipes at the side walls of the tank serve as electrodes³, and a rare earth permanent magnet of diameter $d = 0.5$ cm is located approximately 0.1 - 0.3 cm above the liquid surface. The interaction of the magnetic field (typically 0.04 – 0.09 T in the vertical direction) with the electric current of magnitude $j = 0 - 3$ A results in a horizontal force exerted locally on the fluid in the direction perpendicular to the electric current. Charged particles (ions in dissociated electrolyte solution) drift toward the electrodes such that positive ions move towards the cathode while negative ions move towards the anode. The Lorentz force, $\mathbf{F} = q \mathbf{v} \times \mathbf{B}$, is perpendicular to both the magnetic field and the current. Since the oppositely charged particles move in opposite direction, the resulting force is in the same direction in the horizontal plane for positive and negative ions. If, on the other hand, the particles in addition to drift are carried by the fluid motion, the total force due to this additional velocity component is zero, since positive and negative particles move in the same direction and their total charge is zero. Thus, the electromagnetic force on the fluid does not depend on the motion of the fluid. The force is applied locally to the fluid in a volume where the vertical component of magnetic field is significant. The horizontal extent of this area is of the order of the magnet diameter. The magnetic field decays fast

(approximately as z^{-3}) in the vertical direction. Thus the magnet only effectively induces motion of the fluid in the upper layer.

The magnet is translated along the tank by a stepping motor on a translating stage. The translating stage was constructed from a modified carriage assembly from an old inkjet printer. A simple controller for the stepping motor was designed to allow the translation velocity to be varied in the range $U = 0.2 - 5$ cm/s. The translating stage was leveled to ensure that the distance between the magnet and the surface of water did not change when the magnet is moved along the tank. The polarity of the magnet and the direction of the current were chosen such that the resulting force was applied on the fluid in the same direction as that of the translation of the magnet. This imitates the force applied by a towed body on the fluid (a reaction to the drag force).

A PC video camera placed above the tank is used to record the video sequences or single snapshots of the flow. A camera with adjustable optics and with some software for reducing the horizontal distortions (most cameras have a wide angle objective lens) is preferable for this purpose. Most cameras provide VGA resolution (640 X 480 pixels) and frame rate up to 30 fps which is quite sufficient for the purpose of this experiment. The recorded video can then be analyzed frame by frame using standard software which is usually supplied with the camera. The flow is illuminated by a fluorescent lamp placed below the tank. A food dye can be used to visualize the flows. Several drops of dyed fluid are injected by a pipette along the axis of motion of the object before the object is towed; this is prepared from equal parts of lower and upper layer so that the dyed fluid particles remain at the interface between the layers and form clearly visible patterns of the flow. However, the dye eventually contaminates the fluid. An alternate method is to use a pH-

indicator such as thymol-blue. The solution of this indicator is orange-yellow color in its neutral state and is deep-blue in the basic state. We use a basic solution of the indicator instead of a dye. The water in the upper layer is made slightly acidic, such that it takes only a few minutes after the experiment for the injected blue fluid to become yellow again due to diffusive chemical reaction.

An outline of the steps involved in the experiment follows.

A. Density stratification

The tank is filled with two layers of salt water, each of concentration 30 and 250 g/l and height 0.5 cm. To prepare a two-layer system without mixing the fluids, the heavier layer is first poured into the tank and a sheet of paper is floated on top of it. The less dense fluid is then carefully poured onto the sheet, and the latter is then (very carefully) removed. Another method is to pour the less dense fluid into the tank first, and the heavier fluid can be delivered through a thin tube placed (not too tight) against the bottom of the tank to allow the fluid to spread slowly along the bottom. This second method allows for a very sharp interface between the layers but is somewhat slower to achieve. The two-layer stratification is very stable due to the large density difference between the layers, however, it changes with time due to diffusion of salt. The stratification can be used for approximately one hour, after this period of time it should be replaced.

B. Circular cylinder

The first set of experiments is performed with circular cylinders of different diameters up to 1 cm. No electric current is needed for these experiments. A cylinder is placed vertically in a holder such that the distance between the lower end of the cylinder and the bottom of the tank does not exceed 0.1 – 0.2 cm. By increasing the speed of translation of the cylinder, a transition from the stable regime to a regime with periodic vortex shedding (Karman-Benard vortex street), can be observed. A typical view of the vortex street is shown in Fig. 2. The period of vortex shedding can be measured from a video sequence as time between the events when a vortex of one sign is shedded. Alternatively, the wavelength of the vortex street can be measured from a snapshot of the flow as a distance between the vortices of one sign.

C. Calibration

In the experiments with the magnet it is impossible to estimate the exact value of the force acting in the fluid without exact knowledge of drift velocities of ions. However, an indirect method can be used to obtain the value of this important control parameter. The magnitude of the force is controlled in the experiments by varying the current. If the magnet is stationary, the localized force generates a starting vortex dipole which is a jet with a pair of vortices of opposite sign at its front. Previous studies⁴ show that the distance traveled by a dipole varies with time as

$$L(t) = (3J/4\pi)^{1/3} t^{2/3}. \quad (1)$$

Here, the distance L is measured from the origin (the center of the magnet) to the front of the vortex dipole. The magnet is located approximately at the center of the tank for this calibration. We also assume that the flow is approximately uniform in the vertical direction along the depth of the layer. A drop of dyed fluid is injected under the magnet

before the start of the experiment. Individual frames from the recorded video of the flow can be saved and analyzed using a standard image tool in MS Windows. The position of the cursor in pixels is used to measure the length L at different frames, and a centimeter scale placed under the tank prior to recording the video enables L to be calculated in centimeters. Knowing the video frame rate allows the time between successive frames to be determined. Equation (1) allows us to relate the momentum per unit time per unit depth of the layer, J , to the magnitude of the electric current. L is measured as a function of time for different values of current j . The power law (2/3) dependence is confirmed for each run and the momentum flux, J , is estimated from the slope of the L versus $t^{2/3}$ graph. The calculated values of J can be plotted against current to obtain a linear relation of the form $J = \beta j$, where the coefficient β depends on the particular parameters of the apparatus, namely the distance of the magnet from the surface of water, the magnetization of the magnet, the area of the electrodes and the stratification. In the present work the value of β was in the range 0.3 - 1.5 for J measured in cm^3/s^2 and j in A. After calibrating, the magnet or the electrodes cannot be moved. The calibration procedure should be repeated 2 or 3 times between successive experiments because the value of the coefficient β changes slowly with time due to changes in the stratification. An alternative method to measure the force applied on the fluid requires measurements of velocities in the fluid using a particle image velocimetry (PIV) system such as that described in⁵. The force can then be obtained by integration along a contour around the origin using a so-called flux formula given in⁶.

D. Virtual body

In this part of the experiment a series of runs with different magnet velocities and currents are performed to identify the regimes of the flow. Typical images of the flow are shown in Fig. 3 and 4. When the speed of the magnet is relatively large while the force is relatively weak, the wake behind the magnet is stable and is in the form of a jet (Fig. 3 a) flowing in the direction of motion of the forcing. For even larger values of J the formation of a recirculation in the form of a vortex dipole (Fig. 3 b) is observed under the magnet. The dipole does not remain stable however and starts shedding vortices. As the dipole loses vorticity of one sign it becomes asymmetric and eventually turns in the direction of rotation of the remaining vortex. This results in the formation of a typical Karman-Benard vortex street (Fig. 4). When the forcing is increased even further the nonlinearity of the flow increases to such an extent that the forcing produces strong compact vortex dipoles which then eject in different directions quasiperiodically without forming an organized vortex street. When the flow forms a regular vortex street, the wavelength of the street and the period of vortex shedding can be measured in the same manner as that described for the experiments with a cylinder.

III. DATA ANALYSIS

The wavelength of the vortex street, λ is measured for the cylinder wakes and those generated by a virtual body. The frequency of the vortex shedding can then be introduced as $f = U / \lambda$. In order to better understand the behavior of f for different values of the control parameters involved in the problem, it is useful to first perform a simple dimensional analysis. For the cylinder, frequency f depends on a set of three dimensional quantities including kinematic viscosity ν , velocity U and the diameter of the cylinder d .

The dependence which has to be determined in our experiments can be written in dimensionless form as

$$\text{Ro} = \Phi(\text{Re}) \quad (2)$$

where $\text{Ro} = fd^2/\nu$ is the so-called Roshko number and Φ is an unknown function of the Reynolds number $\text{Re} = dU/\nu$. Roshko number is a dimensionless of frequency while the Reynolds number characterizes the ratio of the nonlinearity to dissipation in the flow. A large number of previous experiments^{7,8} demonstrate linear dependence in the form

$$\text{Ro} = 0.212(\text{Re} - 21.2) \quad (3)$$

for moderate values of the Reynolds number $\text{Re} = 50 - 180$. This dependence can be verified in our experiments. For a virtual body however, there is one additional dimensional parameter, namely the magnitude of the force J ($[J]=L^3/T^2$). Note that the force applied by the cylinder on the fluid is not an independent parameter but rather depends on the control parameters of the flow

$$J = C_D dU^2 / 2, \quad (4)$$

where C_D is the drag coefficient dependent on the Reynolds number of the flow. The plot of C_D as a function of Re can be found in many textbooks on fluid dynamics. When J is a free parameter, the dimensionless dependence can be written in a form:

$$\text{Ro} = \Phi(\text{Re}, \Pi_d), \quad (5)$$

where an additional dimensionless parameter is $\Pi_d = J/(dU^2)$. In the experiments with a virtual body, two of the original dimensional parameters namely U and J can be varied (the diameter of the magnet is fixed) and the regimes of the flow can be observed. The diagram of regimes where different symbols indicate the regimes (stable jet, vortex street, vortex dipole) can then be plotted in the space of dimensionless parameters Re and Π_d .

After this investigation of regimes is performed it is desirable to simplify the problem and to reduce it to that for a circular cylinder. For this purpose the current in the apparatus can be adjusted such that the dependence (4) is satisfied and the virtual body imitates a real cylinder. Note that the values of the Reynolds number should also be close to those for the cylinder flow. The dependence of Ro versus Re can then be compared to that for the circular cylinder.

IV. CONCLUSION

We have presented a laboratory experiment to study a hydrodynamical phenomenon of periodic vortex shedding. The subject of this experiment is a complex (nonlinear) fluid flow. However the experiment does not require special knowledge of fluid dynamics. It rather involves general physical principles such as dimensional analysis and conservation laws. The method is based on electromagnetic method of generating the flows. This method is not routine and although simple is used in advanced research.

The experiment described in this paper can be easily downgraded for use in an entry level or intermediate undergraduate laboratory. In this case the students only perform the part where the cylinders are towed and compare their results with those of classical experiments. Figure 5 shows the linear relationship between Roshko number and Reynolds number for cylinders of diameter 0.3 – 1.0 cm. In this case, the bottom layer of fluid consisted of salt water (250 g/l), while the upper layer was fresh water only (we don't need to apply current in this case).

The experiment can also be modified for more advanced level. Two magnets of opposite polarity can be used to imitate self-propelled objects such as swimming microorganisms, fish or submarines (however, strictly speaking only the laminar wakes with small or moderate values of the Reynolds number can be modeled using this setup). Self-propelled bodies generate zero-momentum flow when moving with constant speed because they have to transfer momentum to the fluid in the direction opposite to that of their motion to generate the thrust force. If a self-propelled body moves with constant speed, the drag and thrust forces are of equal magnitude and of opposite directions. The drag force is applied near the front of the body while the thrust force is usually generated near the rear end. Thus, these forces are separated by a distance which is approximately equal to the length of the body. The same is also valid for the corresponding reaction forces which act on the fluid. A rich variety of regimes of the flow can be observed in the experiments with two magnets. The experiment described in this paper can also include a PIV system⁵. This will allow measurements of velocity in the flows to be made. In particular the profile of velocity in the wake behind a cylinder or a magnet can be measured and the volume flux Q in the wake can be derived. This volume flux is directly related to either the reaction to the drag force applied by the cylinder or the force applied by the magnet via $J = UQ$.

ACKNOWLEDGEMENTS

The research reported in this paper has been supported by the Natural Sciences and Engineering Research Council of Canada under grants 300805-04 and 227192-04.

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Figure captions:

Figure 1. Sketch of the experimental set-up.

Figure 2. Karman-Benard vortex street behind a circular cylinder of diameter $d = 0.478$ cm moving with velocity $U = 1.74$ cm/s, $Re = 83$.

Figure 3. Images of the flow generated by a magnet (virtual body) for different regimes:

(a) stable jet and (b) vortex dipole with a wake behind it. Experimental parameters:

(a) $U = 1.45$ cm/s, $J = 0.8$ cm³/s² and (b) $U = 0.29$ cm/s, $J = 0.7$ cm³/s².

Figure 4. Sequence of video frames showing different phases of a regular vortex shedding regime. Experimental parameters: $U = 0.58$ cm/s, $J = 4.25$ cm³/s².

Figure 5. Experimental results showing the linear relation between Roshko number and Reynolds number. A least squares fit to the data gives $Ro = 0.20(Re - 21)$ consistent with the prediction of Eq (3).

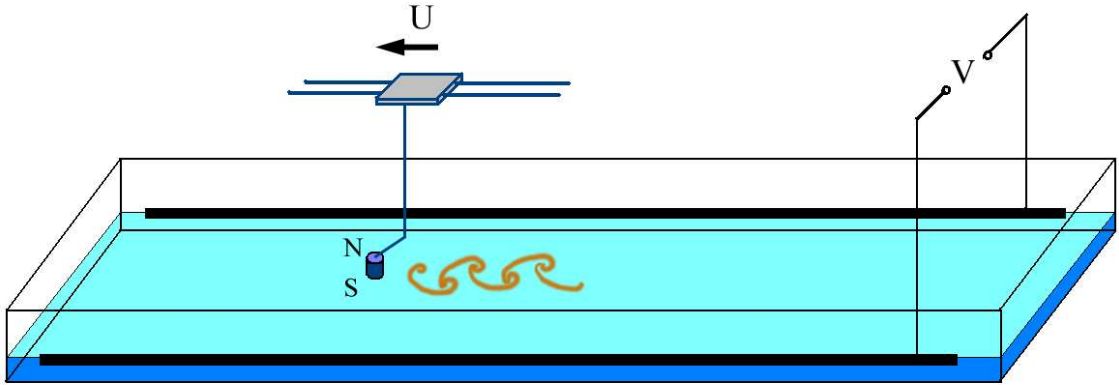


Figure 1.

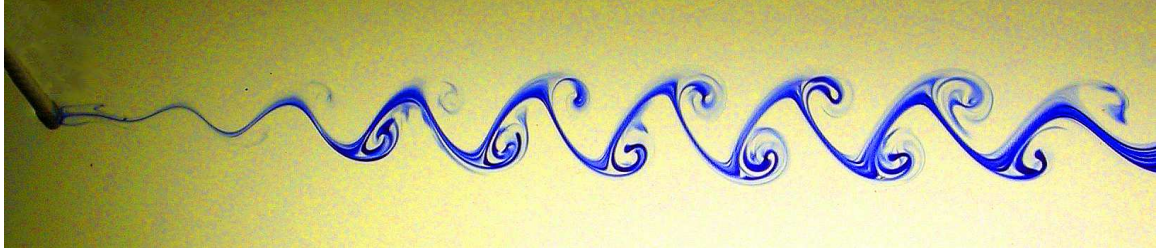


Figure 2.

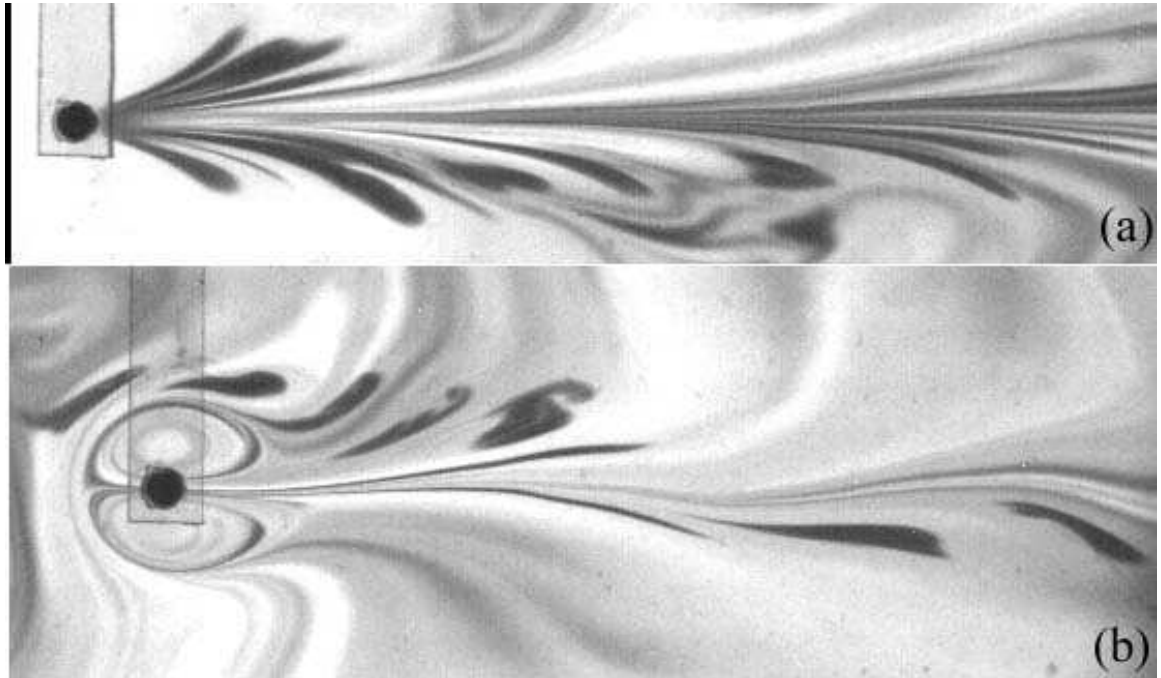


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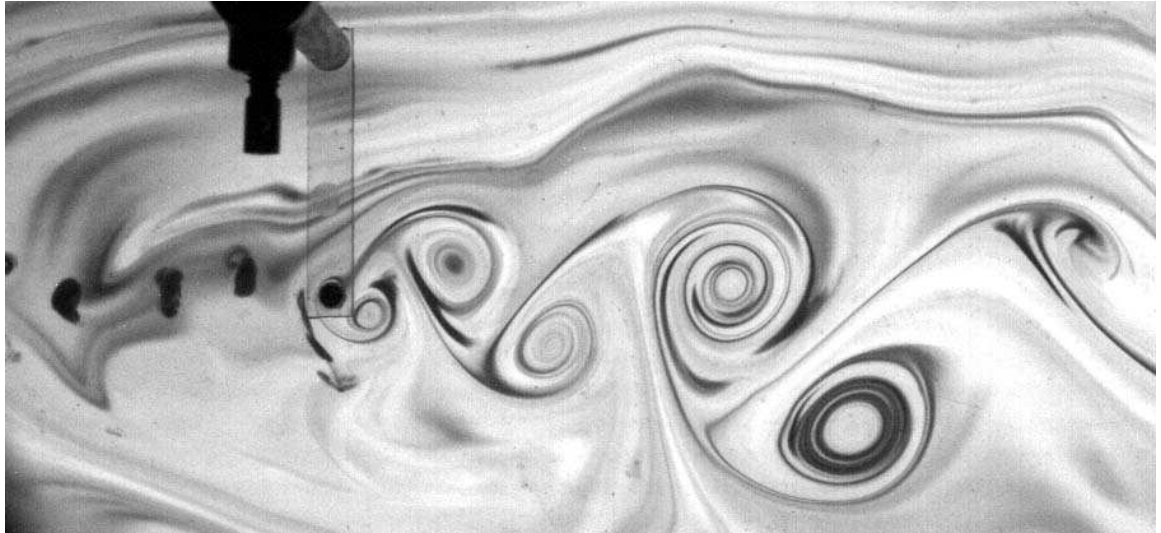


Figure 4.

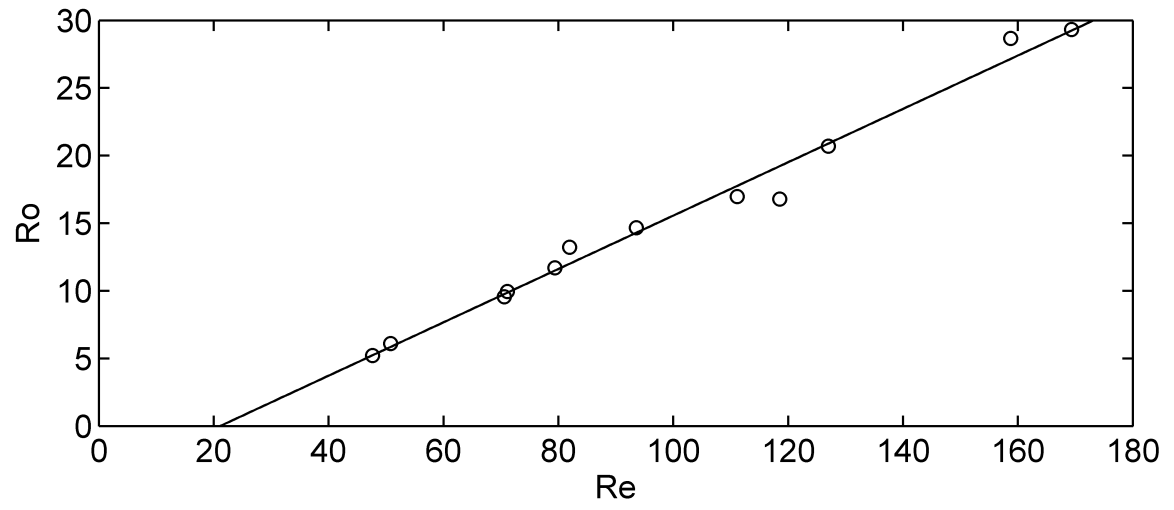


Figure 5.