# Reply to Comment on the paper: On breaking internal waves over the sill in Knight Inlet

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The results of the numerical simulations described in Afanasyev and Peltier (2001) clearly demonstrate that the breaking of a stationary internal wave induced by stratified flow over the topography of the Knight Inlet sill may cause the development of an accelerated downslope flow similar to that described by Farmer and Armi (1999) and to the downslope windstorms observed in the atmosphere. In their comments on the results of our numerical simulations, Farmer and Armi argue on the basis of their oceanographic data that the well mixed intermediate layer water, which is essential for the formation of the accelerated flow, is generated by small scale shear instability rather than by the breaking internal wave. These instabilities were not observed in our simulations although the spatial resolution of the numerical model would have allowed us to reproduce motions on this small scale if they were supported dynamically. Clearly the inflow velocity profile could be very important for the development of favorable conditions for the instability to occur. We employed uniform inflow velocity over the entire depth of the water column in our analyses. In better accord with the vertical profile of horizontal velocity of the actual tidally forced flow over the sill would be a profile with weak reverse flow in the upper layer above the halocline such as is typical for the circulation in a highly stratified estuary. This would impose a shear layer in the incident flow that would be enhanced above the sill and perhaps be subject to Kelvin-Helmholtz instability. Through this simple modification to the profile of horizontal velocity it is entirely possible that the level of mixing associated with the small scale shear instability could be sufficiently enhanced that breaking of the topographically forced internal wave would not be required to match the observations that have been discussed by Farmer and Armi (1999, 2001). Analyses of this kind will be reported elsewhere.

A further interesting question to which our simulations provide no answer concerns the effect of boundary layer separation on the dynamics of the flow (see the discussion of this issue in Cummins, 2000). For the simulations we have performed, the boundary layer separation that does occur initially during the ebb phase of the tidal cycle in the actual Knight Inlet flow is absent from the numerical results. The effects due to boundary layer detachment are certainly of great interest and deserve further attention. Our numerical model has to be modified to allow incorporation of the required no-slip boundary condition and the viscous boundary layer which is thereby formed. This will require implementation of triple deck theory to properly represent the process of boundary layer separation (e.g. Sychev 1990). Results obtained on the basis of analyses of flows with different inflow velocity profiles which allow the development of the instability in the halocline as well as flows which incorporate boundary layer separation will be reported in due course.

As correctly noted by Farmer and Armi (2001) our brief comment (Afanasyev and Peltier, 2001) on their hydraulics based analysis was based on equations of motion derived on the basis of the assumption that the reduced gravity $g'$ was independent of $x$ (Baines, 1995). Although the modified hydraulic approach that they employ is better suited to the present circumstance, which involves intense mixing, this approach is questionable in detail and is not essential for comparison of their Knight Inlet observations with the results of our numerical simulations. It is nevertheless useful to more fully develop our own version of the hydraulic analysis in order to make contact with Farmer and Armi (1999) and other work on exchange flows, as well as to fully respond to their expressions of concern.

Farmer and Armi (1999) employed a Bernoulli integral for a single layer in their hydraulic analysis, the only difference from the conventional treatment being that a horizontally varying reduced gravitational acceleration, defined as $g'(x) = g\frac{\Delta\rho(x)}{\rho}$, was assumed to act upon this layer. The original Bernoulli integral cannot be obtained, however, by integrating the equation of motion along the streamline in the lower layer when the gravitational force is not expressible as the gradient of a potential, which is the case when the gravitational acceleration depends on $x$. Here we choose to employ the usual hydraulic principles, namely those of mass and momentum conservation applied to the lower layer, in order to make all of the assumptions explicit. A horizontally varying pressure $p_0(x)$ due to the presence of the upper layer is assumed to act upon the lower layer. Making use of the assumed hydrostaticity of the flow, one can express $p_0(x)$ in the following form

$$p_0(x) = -(\rho - \Delta\rho(x))g\eta,$$

where $\eta$ is the displacement of the interface. The horizontal pressure gradient in the lower layer is then given by

$$\frac{1}{\rho}\frac{\partial p}{\partial x} = \frac{\partial p_0}{\partial x} + \rho g\frac{\partial \eta}{\partial x} = \frac{\partial g'\eta}{\partial x}.$$

The dynamics of one-layer flow are then described by the equations of motion and continuity (see e.g. Baines 1995) modified to include the horizontally varying pressure as follows:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -\frac{\partial g'\eta}{\partial x}, \qquad (1)$$

$$\frac{\partial y}{\partial t} + \frac{\partial}{\partial x}(yu) = 0, \tag{2}$$

where $y$ is the thickness of the layer. For stationary flow, equations (1) and (2) may be directly integrated, yielding:

$$\frac{u^2}{2} + g'\eta = \frac{u_c^2}{2} + g'_c\eta_c \tag{3}$$

$$uy = u_c y_c \tag{4}$$

Here the subscript $c$ is employed to identify the value of a variable at the crest of the sill. If one introduces a local Froude number, defined by

$$Fr^2 = \frac{u^2}{g'y}, \tag{5}$$

(3) and (4) then give

$$\frac{Fr_c^2}{2\tilde{y}^2} + \tilde{g}'\tilde{\eta} = \frac{Fr_c^2}{2} + \tilde{\eta}_c. \tag{6}$$

In (5), $y$ and $\eta$ are normalized by the thickness $y_c$ of the layer at the crest while $g'$ is normalized by $g'_c$. We may define the nondimensional interface displacement

$$\tilde{\eta} = \tilde{y} + \tilde{h} - \tilde{y}_0, \tag{7}$$

where the normalized topography $\tilde{h} = (h_c - h)/y_c$ and $\tilde{y}_0$ is the initial thickness of the layer. The usual hydraulic condition $Fr_c^2 = 1$ then delivers good agreement with the observational data provided by Farmer and Armi (1999). If one furthermore assumes $\tilde{y}_0 = 3/2$ in (7), then equation (6) takes the same form as that employed in their paper. The analysis presented here makes clear the assumptions on which the equation depends.

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