

On the frontal collision of two round jets in water

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In our experiment two laminar round jets collide in water forming a zero-momentum toroidal vortex and this flow is modeled theoretically. First, the linearized time dependent basic solution for the starting round jet is derived in a straightforward manner. Then a superposition of these solutions is used to model the frontal collision of two round jets. The resulting flow patterns are calculated and compared with the experiments. The comparison shows good qualitative agreement. © 2003 American Institute of Physics. [DOI: 10.1063/1.1613644]

I. INTRODUCTION

Compact vortex structures in a homogeneous fluid are a well-known phenomenon. They are easily formed in water, water-glycerol solutions and other fluids when a force is applied locally to some volume of fluid. If a force acts for a short period of time, vortex rings are produced (see, e.g., photos 7.2.2 and 7.2.3 in Batchelor¹). If the force acts continuously, a starting jet with a spherical (mushroom-like) vortex at its leading edge is generated (see, e.g., Fig. 1 in Voropayev *et al.*²). Such starting jets (round and planar) arising in a viscous fluid under the action of a localized momentum source were previously studied both experimentally and theoretically by many authors (see, e.g., Voropayev *et al.*,² Abramovich, Solan,³ Sozou and Pickering,⁴ Sozou,⁵ Voropayev,⁶ Cantwell,⁷ Voropayev and Afanasyev⁸).

Below we consider the more complicated case shown in Fig. 1. In this experiment two laminar round jets start moving in water against each other [Fig. 1(a)] and collide, forming a toroidal zero-momentum vortex [Fig. 1(b)]. To describe such flows theoretically a simplified theoretical model is proposed. First, the linearized time dependent basic solution for a starting round jet is derived in a straightforward manner. Then a superposition of these solutions is used to model the frontal collision of two round jets. Finally, using the analytical model the resulting flow patterns are calculated and compared with experiments.

The standard approach in finding simplified solutions for starting (round or planar) jets induced by concentrated momentum source in a viscous fluid is as follows (see Sozou, Pickering,⁴ Sozou,⁵ Cantwell⁷). First, a general linear solution for a momentum source should be derived. Then, an integral boundary condition for the conservation of momen-

tum in a control volume is used to find a particular solution. This is not a simple procedure because it includes the integration of the general time dependent solution (with unknown constants) over the control volume. In addition, this approach does not permit a particular solution to be found for flows with zero net momentum, e.g., flows induced by a force doublet⁹ or more complex forcing. For zero momentum flows an integral boundary condition for the conservation of momentum becomes useless and there are no proper conservation integrals at our disposal to find a particular solution. Taking this into account we use a much simpler approach, which permits us to find linearized particular basic solutions for an arbitrary forcing. In this approach a particular solution for the pressure is derived first and then this solution is used in finding a particular solution for the velocity field.

In our previous studies with quasi-planar flows generated in a thin layer of stratified fluid, it was shown (Voropayev *et al.*,² Voropayev, Afanasyev,⁸ Afanasyev *et al.*⁹) that planar linearized solutions described the main features of the real flow patterns (as visualized by the passive tracer) surprisingly well. Dipoles induced by a momentum source as well as quadrupoles formed by two colliding jets were reproduced experimentally and modeled theoretically for the low and moderate values of the Reynolds number. Herein we report on the results of experiments in which two laminar round jets collide in a homogeneous fluid. This system is in fact an axisymmetric analog of the planar vortex quadrupole. Such an experiment in a homogeneous fluid was somewhat more challenging than similar (quasi-two-dimensional) experiments in a stratified fluid because it is more difficult to avoid disturbances (e.g., due to small density fluctuations), which disrupt the three-dimensional flow in a homogeneous fluid. The specific technique described in Sec. III was used to avoid this problem. This technique involves the application of the pH-indicator for the visualization of the flows.

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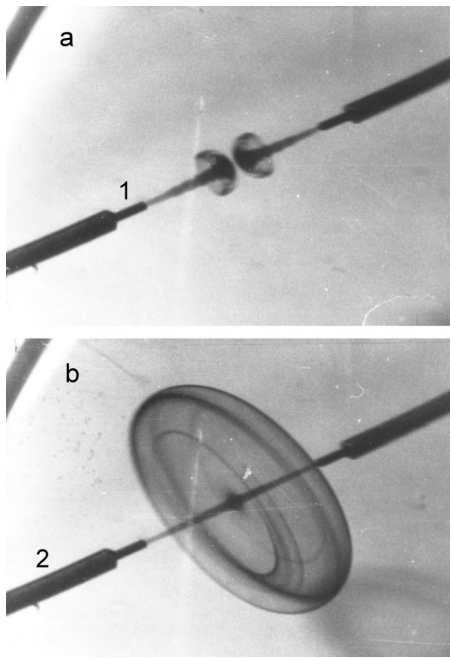


FIG. 1. Head-on collision (a) of two round laminar jets in water and formation (b) of a zero-momentum vortex ring-like structure. Oblique view: (1)—round nozzle; (2)—supporting frame.

II. BASIC SOLUTION

Consider a three-dimensional unsteady flow of a viscous incompressible fluid induced by an axisymmetric localized source of momentum. The fluid is unbounded and is initially at rest. Consider the case of a point momentum source which starts to act at time $t=0$ and thereafter exerts on the fluid a kinematic momentum flux (force per unit mass) equal to $\mathbf{J} = \text{const}$. (The other cases of practical interest, for example, an impulsive momentum source or the more complicated forcing of a force doublet, may be considered in a similar manner.) The balance of momentum and condition of mass conservation for a fluid with such singularity at the origin $|\mathbf{x}|=0$ can be presented in the form

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \nabla) \mathbf{u} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u} + H(t) \mathbf{J} \delta(\mathbf{x}), \quad (1)$$

$$\nabla \mathbf{u} = 0, \quad (2)$$

where $\mathbf{u}=(u,v,w)$ is the velocity vector, p the pressure, ρ the density, ν the kinematic viscosity, \mathbf{x} the position vector, $\delta(\mathbf{x})$ the Dirac delta function, $H(t)$ the Heaviside step function, and ∇ the gradient operator.

We are looking for a time dependent linear solution (first order approximation). Neglecting the nonlinear term of (1), one obtains the Stokes equations. Taking the divergence of these equations and employing the fact that in three-dimensions $4\pi\delta(\mathbf{x}) = -\nabla^2(1/|\mathbf{x}|)$, one arrives at the Poisson equation,

$$\nabla^2 \left(\frac{1}{\rho} p + H(t) \frac{\mathbf{J}}{4\pi} \nabla \frac{1}{|\mathbf{x}|} \right) = 0, \quad (3)$$

from which a particular solution for the pressure follows:

$$\frac{1}{\rho} p = H(t) \frac{\mathbf{J} \mathbf{n}}{4\pi|\mathbf{x}|^2} = H(t) \frac{J \cos \theta}{4\pi r^2}, \quad (4)$$

where $\mathbf{n}=\mathbf{x}/|\mathbf{x}|$, $J=|\mathbf{J}|$, $r=|\mathbf{x}|$, the spherical polar coordinates (r, θ, ϕ) are used for the last term and the source is directed at $\theta=0$ in this coordinate system. The solution (4) gives the proper condition, which relates the flow characteristics with the forcing amplitude, and (4) is used below in finding a particular solution for the velocity.

As can be seen from (4), at $t>0$ the pressure instantaneously arises in the fluid and does not change with time. To find the first order velocity field one must solve (1), where the pressure is now known and a nonlinear term is neglected. We are looking for an axisymmetric (the source of motion is axisymmetric) solution: $\mathbf{u}=(u,v,0)$ in spherical polar coordinates and the motion does not depend on the azimuthal coordinate ϕ . A stream function Ψ is then introduced in a standard form as

$$u = \frac{1}{r^2 \sin \theta} \frac{\partial \Psi}{\partial \theta}, \quad (5)$$

$$v = -\frac{1}{r \sin \theta} \frac{\partial \Psi}{\partial r}. \quad (6)$$

The substitution of (5) into the radial component of the linearized momentum equation (1) gives the equation (in spherical polar coordinates) in terms of pressure and stream function,

$$\frac{1}{r^2 \sin \theta} \frac{\partial}{\partial t} \frac{\partial \Psi}{\partial \theta} = -\frac{1}{\rho} \frac{\partial p}{\partial r} + \frac{\nu}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\frac{\partial^2}{\partial r^2} + \frac{\sin \theta}{r^2} \frac{\partial}{\partial \theta} \left(\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \right) \right) \Psi. \quad (7)$$

We are looking for a similarity solution (only such solutions can be found analytically). The expression in terms of the similarity variable $\eta=r/2\sqrt{\nu t}$ for the stream function can be obtained on dimensional grounds, for example, as

$$\Psi = \frac{Jt^{1/2}}{\nu^{1/2}} \Psi^*(\eta) \sin^2 \theta, \quad \eta = r/2\sqrt{\nu t}. \quad (8)$$

The substitution of (4) and (8) into (7) renders an ordinary differential equation,

$$2\eta^2 \frac{d^2}{d\eta^2} \Psi^* + 4\eta^3 \frac{d}{d\eta} \Psi^* - 4(1+\eta^2) \Psi^* = -\frac{\eta}{\pi}, \quad (9)$$

for which the general solution can be presented in the form

$$\Psi^* = \frac{1}{8\pi\eta} + C_1 \left(\eta - \frac{1}{2\eta} \right) - C_2 \frac{\sqrt{\pi}}{4\eta} \left(\frac{2\eta}{\sqrt{\pi}} e^{-\eta^2} + (2\eta^2 - 1) \text{erf}(\eta) \right), \quad (10)$$

where C_1, C_2 are the constants of integration, $\eta=r/2\sqrt{\nu t}$ and $\text{erf}(\eta)=(2/\sqrt{\pi})\int_0^\eta e^{-x^2} dx$.

To find a particular solution of (10) one needs to calculate C_1 and C_2 . The condition that the fluid is initially at rest (and is at rest at infinity for any time) gives $\Psi^*(\infty)=0$, and

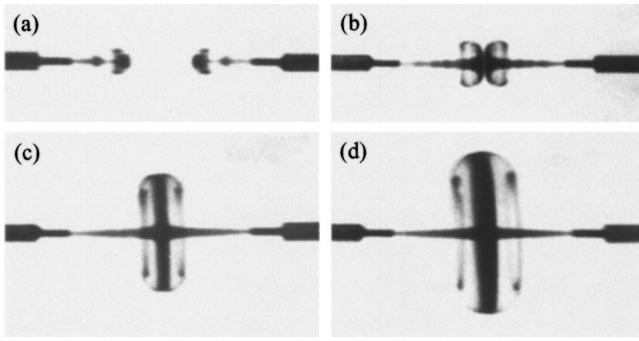


FIG. 2. A succession of side view photographs showing the frontal collision of two round jets in the experiment and the formation of a toroidal (vortex ring-like) structure with zero net momentum. Fluid—distilled water; nozzle diameter $d=0.05$ cm, the distance between nozzles $x_0=1.6$ cm, $J=0.36$ cm⁴ s⁻¹ and $Re=J^{1/2}/\nu\approx 60$. Visualization—pH-indicator thymol blue. Approximate time from the beginning of the experiment: $t\approx 1$ (a), 5 (b) 20 (c), 30 s (d).

using this one finds $C_1=C_2\sqrt{\pi}/2$. From general principles (Van Dike¹⁰) we require the minimum possible singularity at the origin ($\eta=0$), and this gives $C_2=1/2\pi^{3/2}$. Thus, a particular solution (in dimensional form) is given by

$$\Psi = \frac{Jt^{1/2}}{8\pi\nu^{1/2}\eta} \left(2\eta^2 - \frac{2\eta}{\sqrt{\pi}} e^{-\eta^2} - (2\eta^2 - 1)\text{erf}(\eta) \right) \sin^2 \theta, \quad \eta = r/2\sqrt{vt}. \tag{11}$$

For Ψ given by (11), the velocity components are

$$u(r, \theta, t) = \frac{J}{2\pi\eta^3(4\nu)^{3/2}t^{1/2}} \left(2\eta^2 - \frac{2\eta}{\sqrt{\pi}} e^{-\eta^2} + (1 - 2\eta^2)\text{erf}(\eta) \right) \cos \theta, \tag{12}$$

$$v(r, \theta, t) = \frac{J}{4\pi\eta^3(4\nu)^{3/2}t^{1/2}} \left(-2\eta^2 - \frac{2\eta}{\sqrt{\pi}} e^{-\eta^2} + (1 + 2\eta^2)\text{erf}(\eta) \right) \sin \theta. \tag{13}$$

Thus, using our simplified method we recover the basic solution that was derived using a much more complicated approach in Sozou, Pickering,⁴ Sozou,⁵ Cantwell.⁷

III. COMPARISON WITH THE EXPERIMENT

A general view of colliding round jets is shown in Fig. 1. For a better comparison with calculations given below, the process of frontal collision of two round jets is shown in greater detail in a succession of side view photographs in Fig. 2. In this experiment two round jets of equal intensities J are generated from two small round nozzles (diameter $d=0.05$ cm) separated by a distance $x_0(=1.6$ cm). The working fluid is distilled water ($\rho=1$ g cm⁻³, $\nu=0.01$ cm² s⁻¹) and flow is visualized by pH-indicator thymol blue. (In contrast to the usual dyes, this method does not change the water

density;² our attempts with usual dyes failed because the smallest differences in density cause the rapid loss of the flow symmetry.) By measuring the volume flux rate q from each nozzle, the value of J may be accurately estimated as (see, e.g., Batchelor,¹ Voropayev *et al.*²) $J=4q^2/\pi d^2$, and this gives $J\approx 0.34$ cm⁴ s⁻² and $Re=J^{1/2}/\nu\approx 60$.

As can be seen in Fig. 2, initially each jet develops independently generating a starting (mushroom-like) spherical vortex at its leading edge [Fig. 2(a)]. With time [Fig. 2(b)] these vortices increase in size and collide, forming a non-propagating toroidal (vortex ring-like) structure [Fig. 2(c)]. However, in contrast to a vortex ring, which has nonzero momentum, this toroidal vortex has zero net momentum and does not move. At later times [Fig. 2(d)] the general structure of the resulting vortex does not change significantly and the vortex slowly increases in size.

In the experiments a colored passive tracer is used to visualize the flow pattern. If the velocity components U, V are known [Cartesian velocity components (U, V) and Cartesian coordinates (x, y), where the x -axis coincides with the polar axis $\theta=0$, are used below in calculations], then integrating the equations of motion for marked particles,

$$\frac{dx}{dt} = U, \quad \frac{dy}{dt} = V, \tag{14}$$

it is possible to calculate the distributions of the marked particles at different times and compare these distributions with the colored water patterns in a real flow.

To model the flow theoretically, the appropriate superposition of two velocity fields induced by two momentum sources was used,

$$U = u^*(x, y, t) - u^*(x + x_0, y, t), \tag{15}$$

$$V = v^*(x, y, t) - v^*(x + x_0, y, t),$$

where the velocity components (12), (13) in spherical polar coordinates are used to calculate the Cartesian velocity components u^*, v^* in (15). The second terms on the right hand sides of (15) are negative and represent the velocity induced by a second source of momentum located at $(x, y)=(x_0, 0)$ and acting in the opposite direction to the first source located at the origin of the coordinate system (see Fig. 3). The system of ordinary differential equations (14), where the right hand side is known can therefore be easily solved numerically using standard solvers available in various applied mathematical software packages. It is important however to specify initial conditions for the marked particles such that the calculations closely reproduce the experiment where the dyed fluid was injected continuously.

In our calculations the values of the dimensional control parameters as well as the time were the same as that in the experiment shown in Fig. 2. Each marked particle started its motion at time t_0 from position (r_0, θ_0) . To model the continuous injection of dyed fluid during the experiment, we launched 1600 particles continuously with time interval 0.05 s from equally spaced points in the interval $-\pi/2 < \theta_0 < \pi/2$ on the arc of radius $r_0=d/2$ (d is the nozzle diameter used in the experiment). The endpoints of the trajectories of all the injected particles at any time t gave the outline of the

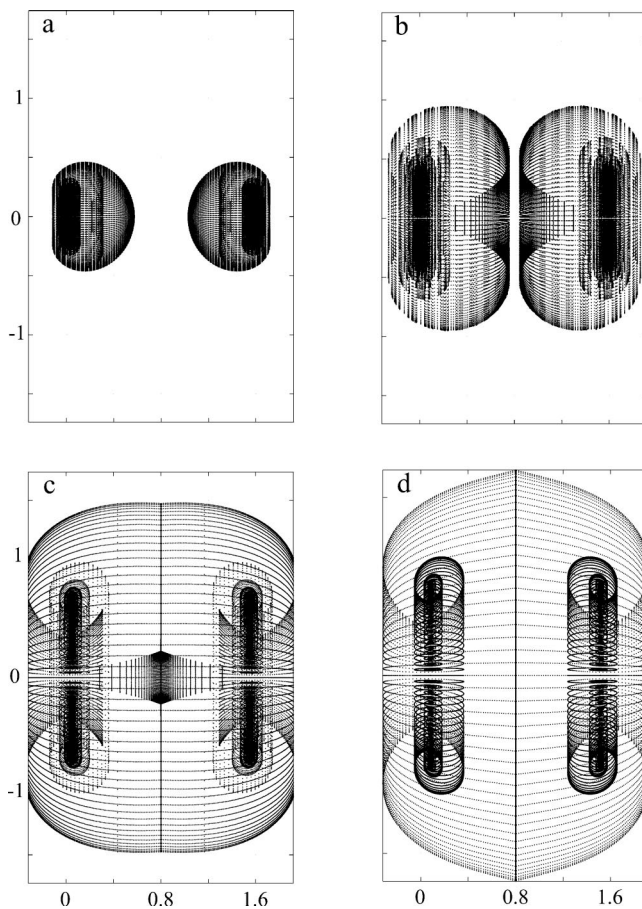


FIG. 3. A succession of calculated side view images showing the frontal collision of two round jets and the formation of a toroidal (vortex ring-like) structure with zero net momentum. The values of external parameters in the theoretical model are the same as in the experiment shown in Fig. 2: $\rho = 1 \text{ g cm}^{-3}$, $\nu = 0.01 \text{ cm}^2 \text{ s}^{-1}$, $J = 0.36 \text{ cm}^4 \text{ c}^{-2}$, $x_0 = 1.6 \text{ cm}$ and $\text{Re} = J^{1/2}/\nu = 60$. Time $t = 1$ (a), 5 (b), 20 (c), 30 s (d). The motion of marked particles is simulated using the appropriate superposition of two basic solutions. The scale is in cm.

front of the dyed fluid (“hat of the mushroom”). The second series of particles was launched continuously from the points $r_0 = d/2$, $\theta_0 = \pm \pi/2$ in time intervals 0.0001 s. These particles gave the outline of the side surface of the dyed fluid (“leg of the mushroom”). The initial time was $t_0 = 0.01 \text{ s}$ such that $\eta_0 = \eta(t=0) \gg 1$.

The three-dimensional distribution of the marked fluid particles for the considered axisymmetric flow can then be obtained by rotating the calculated planar distributions about the x -axis. A sequence of images in Fig. 3 shows the side view of the flow for different times. In the initial stages of the flow evolution the two jets, one induced by each momentum source, develop almost independently [Fig. 3(a)] forming two mushroom-like frontal vortices. As time progresses, the jets begin to interact and finally collide [Fig. 3(b)], forming a toroidal vortex with zero net momentum [Fig. 3(c)]. At latter times [Fig. 3(d)] the general structure of the resulting vortex does not change significantly and the vortex only increases slowly in size in a similar manner.

Strikingly, but in spite of great simplifications, the linearized model (Fig. 3) correctly reproduces qualitatively practically all of the details of the real (Fig. 2) jets’ collision

and the formation of the resulting toroidal vortex. A quantitative comparison shows, however, that the calculated flow patterns are somewhat broader than that observed in the experiment. But it is hardly possible to expect better agreement between the experiment conducted at moderate Reynolds number, $\text{Re} \approx 60$, and simplified model, which was used for the same Re as in the experiment, but, strictly speaking, is valid only for $\text{Re} \ll 1$. By decreasing Re in the experiment and in the model, much better quantitative agreement with the theory may be obtained but at smaller Re such experiments become complicated: it is too difficult to control very small flux rates and make them exactly equal for both nozzles.

IV. CONCLUSIONS

In experiments conducted at moderate Reynolds numbers two round jets from small nozzles collide in water, forming a toroidal vortex with zero net momentum. This flow was modeled theoretically using the proper superposition of two linearized solutions for a starting jet induced by a concentrated force in a viscous fluid. Although at first glance such unsteady linear solutions do not appear to describe the dynamics of colliding jets in a real fluid where nonlinear effects may be thought of as the most important, a further comparison shows good qualitative agreement between the theory predictions and observations.

Finally, note that for small forcing amplitude the solution of (1) can be expanded in a series using the nondimensional forcing amplitude $\text{Re} = J^{1/2}/\nu$ as a small parameter. Then the first order linear solution can be used to calculate the nonlinear term in (1) and the second order nonlinear solution can be found. This is a rather tiresome procedure and some examples can be found in Voropayev *et al.*,¹¹ Cantwell and Rott,¹² and Voropayev and Afanasyev¹³). Some interesting features of the weakly nonlinear solution as compared to those of the linear solution are worth noting. In particular, the weakly nonlinear solution allows the frontal vortex to drift forward in the direction of the action of the source of motion. In contrast, the linear solution describes only the homogeneous expansion of the vortex relative to the point of origin. The nonlinearity therefore causes translational motion of the frontal vortex and makes the jet narrower. However, the superposition of the weakly nonlinear solutions cannot be used to describe the jets’ collision; only for linear solutions is such a superposition possible.

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