Jacobi method for eigenvalues and eigenvectors of real symmetric matrix

In[730]:=
```
Remove["Global`*"]
```

Construct Given’s rotation matrix.

In[731]:=
```
G[l_, m_, c_, s_, n_] :=
 Table[If[i == j, If[i == l && j == l || i == m && j == m, c, 1], If[i == l && j == l,
    c, If[i == l && j == m, -s, If[i == m && j == l, s, 0]]]], {i, 1, n}, {j, 1, n}]
```

In[732]:=
```
(G[2, 4, c, s, 5]) // MatrixForm
```

Out[732]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & c & 0 & -s & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & s & 0 & c & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[733]:=
```
(G[1, 2, c, s, 3]) // MatrixForm
```

Out[733]//MatrixForm=

$$\begin{pmatrix} c & -s & 0 \\ s & c & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Define rotation angle θ that will zero the (l,m) element of a matrix under a similarity transformation.

In[734]:=
```
θ[l_, m_] := (Aij = A[[l, m]]; Ajj = A[[m, m]]; Aii = A[[l, l]];
  diff = Ajj - Aii;
  If[diff == 0, π / 4, ArcTan[diff, 2 Aij] / 2])
```

“Random” symmetric matrix.

In[735]:=
```
(A = {{1, 3, 12}, {3, 2, 4}, {12, 4, 7.}}) // MatrixForm
```

Out[735]//MatrixForm=

$$\begin{pmatrix} 1 & 3 & 12 \\ 3 & 2 & 4 \\ 12 & 4 & 7. \end{pmatrix}$$

Initialize eigenvector matrix (rows will be eigenvectors).

In[736]:=
```
eigv = IdentityMatrix[3]
```

Out[736]=
```
{{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}
```

In[737]:=
```
Acopy = A;
```

Largest off-diagonal element is at l=3, m=1

```
In[738]:= l = 3;
          m = 1;
          c = Cos[θ[l, m]]
          s = Sin[θ[l, m]]

Out[740]= 0.615412

Out[741]= 0.788205

In[742]:= (g = G[l, m, c, s, 3]) // MatrixForm
Out[742]//MatrixForm=
```

$$\begin{pmatrix} 0.615412 & 0 & 0.788205 \\ 0 & 1 & 0 \\ -0.788205 & 0 & 0.615412 \end{pmatrix}$$

```
In[743]:= (A = g.A.Transpose[g]) // MatrixForm
          eigv = g.eigv
Out[743]//MatrixForm=
```

$$\begin{pmatrix} 16.3693 & 4.99906 & -8.88178\times10^{-16} \\ 4.99906 & 2. & 0.0970325 \\ -1.77636\times10^{-15} & 0.0970325 & -8.36932 \end{pmatrix}$$

```
Out[744]= {{0.615412, 0., 0.788205}, {0., 1., 0.}, {-0.788205, 0., 0.615412}}
```

Now largest off-diagonal element is at l=2, m=1

```
In[745]:= l = 2;
          m = 1;
          c = Cos[θ[l, m]]
          s = Sin[θ[l, m]]
          (g = G[l, m, c, s, 3]) // MatrixForm

Out[747]= 0.954162

Out[748]= 0.299291

Out[749]//MatrixForm=
```

$$\begin{pmatrix} 0.954162 & 0.299291 & 0 \\ -0.299291 & 0.954162 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
In[750]:= (A = g.A.Transpose[g]) // MatrixForm
          eigv = g.eigv
Out[750]//MatrixForm=
```

$$\begin{pmatrix} 17.9374 & -1.77636\times10^{-15} & 0.029041 \\ -6.38378\times10^{-16} & 0.43195 & 0.0925847 \\ 0.029041 & 0.0925847 & -8.36932 \end{pmatrix}$$

```
Out[751]= {{0.587203, 0.299291, 0.752076},
           {-0.184187, 0.954162, -0.235903}, {-0.788205, 0., 0.615412}}
```

Now largest off - diagonal element is at l = 3, m = 2

In[752]:=
```
l = 3;
m = 2;
c = Cos[θ[l, m]];
s = Sin[θ[l, m]];
(g = G[l, m, c, s, 3]) // MatrixForm;
(A = g.A.Transpose[g]) // MatrixForm
eigv = g.eigv
```

Out[757]//MatrixForm=

$$\begin{pmatrix} 17.9374 & 0.000305445 & 0.0290394 \\ 0.000305445 & 0.432924 & -2.55004 \times 10^{-16} \\ 0.0290394 & 1.38778 \times 10^{-17} & -8.37029 \end{pmatrix}$$

Out[758]= {{0.587203, 0.299291, 0.752076}, {-0.192467, 0.954109, -0.229417}, {-0.786225, -0.0100356, 0.617859}}

In[759]:=
```
l = 3;
m = 1;
c = Cos[θ[l, m]];
s = Sin[θ[l, m]];
(g = G[l, m, c, s, 3]) // MatrixForm;
(A = g.A.Transpose[g]) // MatrixForm
eigv = g.eigv
```

Out[764]//MatrixForm=

$$\begin{pmatrix} 17.9374 & 0.000305445 & 8.56953 \times 10^{-16} \\ 0.000305445 & 0.432924 & -3.37161 \times 10^{-7} \\ 0. & -3.37161 \times 10^{-7} & -8.37032 \end{pmatrix}$$

Out[765]= {{0.586335, 0.29928, 0.752757}, {-0.192467, 0.954109, -0.229417}, {-0.786872, -0.010366, 0.617029}}

In[766]:=
```
l = 2;
m = 1;
c = Cos[θ[l, m]];
s = Sin[θ[l, m]];
(g = G[l, m, c, s, 3]) // MatrixForm;
(A = g.A.Transpose[g]) // MatrixForm
eigv = g.eigv
```

Out[771]//MatrixForm=

$$\begin{pmatrix} 17.9374 & -1.12865 \times 10^{-15} & -5.88244 \times 10^{-12} \\ 1.77877 \times 10^{-20} & 0.432924 & -3.37161 \times 10^{-7} \\ -5.8833 \times 10^{-12} & -3.37161 \times 10^{-7} & -8.37032 \end{pmatrix}$$

Out[772]= {{0.586331, 0.299296, 0.752753}, {-0.192478, 0.954104, -0.22943}, {-0.786872, -0.010366, 0.617029}}

In[773]:=
```
l = 3;
m = 2;
c = Cos[θ[l, m]];
s = Sin[θ[l, m]];
(g = G[l, m, c, s, 3]) // MatrixForm
(A = g.A.Transpose[g]) // MatrixForm
eigv = g.eigv
```

Out[777]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1. & -3.82996\times 10^{-8} \\ 0 & 3.82996\times 10^{-8} & 1. \end{pmatrix}$$

Out[778]//MatrixForm=

$$\begin{pmatrix} 17.9374 & -1.12843\times 10^{-15} & -5.88244\times 10^{-12} \\ 2.43116\times 10^{-19} & 0.432924 & -2.70143\times 10^{-16} \\ -5.8833\times 10^{-12} & 5.29396\times 10^{-23} & -8.37032 \end{pmatrix}$$

Out[779]=
```
{{0.586331, 0.299296, 0.752753},
 {-0.192478, 0.954104, -0.22943}, {-0.786872, -0.0103659, 0.617029}}
```

Givens rotation matrix close to identity. Off-diagonals of transformed matrix are nearly zero. Diagonals should give eigenvalues. Rows of our running g.eigv matrix should be eigenvectors. Let's check against *Mathematica*'s answers.

In[780]:=
```
Eigenvalues[Acopy]
```

Out[780]=
```
{17.9374, -8.37032, 0.432924}
```

In[781]:=
```
Eigenvectors[Acopy]
```

Out[781]=
```
{{0.586331, 0.299296, 0.752753},
 {-0.786872, -0.0103659, 0.617029}, {0.192478, -0.954104, 0.22943}}
```