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A Vortex Dipole in an Intrusion Flow of Density-Stratified Fluid

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Results are given from a laboratory experiment that analyzes the flow regimes induced by a combined mass and momentum source in a viscous density-stratified fluid. It is demonstrated that intrusion and the vortex dipole can be treated as two limit cases of the development of such a flow. An explanation is given for the experimental results, and a criterion is proposed for flow transition from intrusion to dipole (mushroom cloud) flow resulting from changes in the intensity ratio of the mass and momentum sources.

INTRODUCTION

Cases where a foreign water mass enters a surrounding density-stratified fluid and extends out into the fluid through its own equilibrium density level are commonly observed in the ocean. Typical examples include the propagation of melt water from the edges of melting ice or icebergs onto the surface of the salt bearing ocean, the influx of fresh river water into the seas, the flow of salty Mediterranean Sea water into the Atlantic Ocean, the dissemination of mixed fluid spots from the collapse of internal waves, etc. In all these cases, a horizontal momentum is conveyed to the surrounding stratified fluid in addition to the foreign water mass.

Laboratory studies to investigate such flows have commonly been conducted for two (limit) flow regimes when the effect of one of two parameters can be neglected. By injecting a foreign fluid into a surrounding stratified fluid through a vertical tube at a low speed, it is possible to reproduce the axisymmetric intrusions and collapse of mixed spots [1, 2]. In this case, the horizontal momentum conveyed to the surrounding fluid is negligible because of flow symmetry, and the fluid influx intensity becomes the fundamental parameter. On the other hand, when injecting a fluid at a signficiant velocity from a thin horizontal nozzle, one can reproduce different vortex dipoles (mushroom cloud flows) in a stratified fluid [3-6]. In this case, the mass of the fluid released by the source is negligible, and the primary parameter is the momentum conveyed to the surrounding fluid.

Generally two parameters can affect flow dynamics. This leads to the question of what ratio between the two parameters will make it possible for the vortex dipole formed in an intrusion flow to become sufficiently powerful to substantially alter the observed flow pattern and lead to a clearly expressed "mushroom cloud" distribution of a passive impurity in the flow (see Fig. 1) below). An attempt is made to answer this question, based on a simple experiment, and to derive a criterion for the transition of one flow regime to another. This criterion may also be useful in predicting the development of oceanic intrusion flows.

EXPERIMENTAL SCHEME

In principle, the experimental scheme is analogous to that used in [3, 4], and we briefly recall its design here. The experiments were conducted in a basin filled with a density-stratified fluid with a Brunt-Väisälä frequency N roughly constant with depth. The fluid density was altered by changing the salinity of distilled water. A circular horizontal nozzle placed in the fluid column was used as the combined mass and moment source, and a jet of colored fluid was ejected from this source at a fixed and strictly controlled volumetrical control rate Q. The fluid density in the jet was equal to the density of the surrounding fluid at the center of the nozzle. In order to run the experiments several times without disturbing the water in the basin, a thymol blue pH indicator was used in place of an ordinary dye for visualization purposes. Quantitative data were obtained by processing series of flow photographs.

We were guided by the following considerations in selecting the nozzle diameter. The method proposed in [3] make it possible to measure and sustain a constant flow rate in the range $Q = 10^{-2}$–10^{-1} $cm^3 \cdot s^{-1}$ within acceptable accuracy. The nozzle diameter (d = 0.32 cm) was selected experimentally to achieve a clear intrusion flow with low flow rates and a mushroom cloud flow with high flow rates. A typical escape velocity of the fluid from the source was $u \approx 0.5$ $cm \cdot s^{-1}$. In this case, we had a small Richardson number $Ri = N^2d^2/u^2$, and the effect of stratification near the nozzle section could be neglected. For definiteness, we shall assume that the fluid was uniformly ejected by the source in all directions with a characteristic velocity u into a hemisphere of diameter d whose area $S = \pi d^2/2$. We obtain the following estimate for Q: $Q = u\pi d^2/2$. The escaping fluid conveyed the following momentum per unit of time along the

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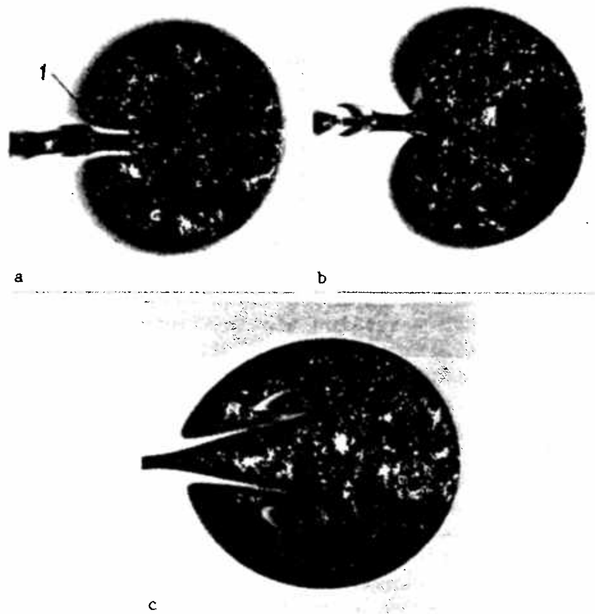


Fig. 1. Photographs of the flows for different source intensities; top view: 1) nozzle (d = 0.32 cm); a) intrusion flow regime: $Q = 1.0\cdot10^{-2}$ cm^3/s, $I = 0.3\cdot10^{-3}$ cm^4/s^2, $N = 3.14$ s^{-1}; b) the onset of a vortex dipole in the intrusion flow accompanied by the influx of the surrounding fluid into the spot; $Q = 3.6\cdot10^{-2}$ cm^3/s, $I = 4.1\cdot10^{-3}$ cm^4/s^2, $N = 3.14$ s^{-1}; c) the dipole flow regime: $Q = 8.3\cdot10^{-2}$ cm^3/s, $I = 21.5\cdot10^{-3}$ cm^4/s^2, $N = 3.14$ s^{-1}.

nozzle axis (the X axis)

$$\rho I = \rho \int_S u u_x dS = \rho u^2 \pi d^2/4,$$

where ρ is the density of the escaping fluid; u_x is the velocity projection onto the X axis. Therefore, for our combined source the specific momentum flux and volumetric flow are related by $I = Q^2/\pi d^2$. Note that with any velocity distribution other than the uniform distribution assumed here the ratio between I and Q will remain unchanged accurate to an insignificant constant of order unity. Therefore, we have a combined mass and momentum source in which the ratio $I/Q \sim Q$ varies with Q.

EXPERIMENTAL RESULTS AND INTERPRETATION

The flow photographs for three different flow rates Q are shown in Fig. 1. In all cases, the flow is a thin plane "spot" of characteristic transverse dimensions D to a first approximation. With small values of Q (Fig. 1a), the spot is entirely filled with the dyed fluid at all times. The entire spot begins to travel slowly forward and expand simultaneously as a result of the influx of fluid from the source. The expansion rate of the spot exceeds its translational velocity, and therefore the trailing edge of the spot slowly stretches out upward, following the flow behind the source. As the flow rate increases, the flow essentially remains unchanged. The translation velocity of the entire spot increases, while the velocity at which the trailing edge of the spot travels upward following the stream behind the source drops. At a certain value $Q = Q_*$, the trailing edge of the spot remains fixed throughout the entire experiment (Fig. 1b), and the entire spot begins forward translational motion and continues to be filled entirely with dyed fluid. There is no clear evident influx of surrounding fluid into the spot. The pattern changes significantly for $Q > Q_*$. The translational velocity of the entire spot exceeds it rate of expansion, and there is intensive absorption of the surrounding undyed fluid. The flow acquires a clearly expressed mushroom cloud shape with a characteristic jet section, and a

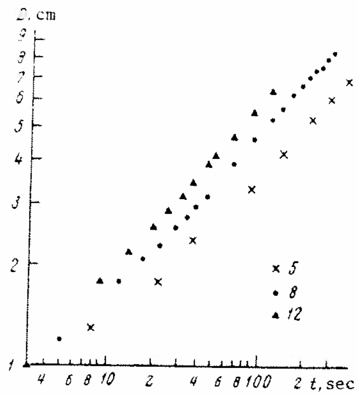


Fig. 2. The characteristic tranverse spot dimensions D plotted as a function of time. The spots 5, 8 and 12 correspond to the values of parameters Q and I listed in Table 1, $N = 2.1\ s^{-1}$.

disk-shaped vortex dipole along the leading edge of the jet (Fig. 1c). The mechanism by which the surrounding fluid is drawn into the mushroom cloud flow has been examined in detail in [4, 5] and is discussed here.

The localized horizontal momentum source produces a flow with a dipole vortex distribution (a vortex dipole) in the viscous density-stratified fluid. The characteristic horizontal flow scale grows over time as $D \sim t^{1/2}$, and the flow develops in a self-similar fashion [4]. On the other hand, the localized mass source in the viscous stratified fluid produces an axisymmetric intrusion flow, which also develops in a self-similar manner and expands horizontally over time proportional to $t^{1/2}$ [2]. On this basis, we expect that the intrusion-mushroom cloud flow excited by the combined bass and momentum source will also be a self-similar flow and will grow in size as $D \sim t^{1/2}$.

We denote by $U_0 = 1/2\ dD/dt$ the characteristic velocity at which the flow expands, and by $U = dx/dt$ the characteristic translational velocity of the flow (x is the coordinate of the flow center). Therefore if the flow develops self-similarly, the ratio of $U/U_0 \equiv A$ will be independent of time. Consequently, there exists an $A = A_*$ such that for $A < A_*$ the intensity of the vortex dipole will not be significant and the flow will develop in an intrusional regime (Fig. 1a). For $A > A_*$, the situation is reversed and the effect of the vortex dipole becomes predominant, and all fluid emitted by the source is contributed to the frontal region of the flow where it produces two characteristic helixes (Fig. 1c). The capture of the surrounding fluid becomes significant in this case, and the flow develops in a mushroom cloud regime. The equality $A_* \gtrsim 1$ is a natural choice for the transition criterion from one flow regime to another. By expressing A through the initial external parameters, we obtain an expression for the transition criterion convenient for practical applications.

The observations reveal that at each instant the flow is a thin plane disk of characteristic transverse dimensions D, which grow over time as $D \sim t^{\alpha}$ where $\alpha = 0.50 \pm 0.03$ (Fig. 2). Disk thickness H is small compared to D and remains virtually unchanged over time. To obtain a quantitative estimate of D, we employ a theoretical relation that has been confirmed by extensive measurements for $A = 0$ (axisymmetric intrusion) [2, 7]:

$$D = C\nu^{-1/10}N^{1/5}Q^{2/5}t^{1/2}, \tag{1}$$

where $C = 0.8$, and ν is the kinematic viscosity coefficient. As is evident from Fig. 3, this relation provides a good description of our measurement results for $A < A_*$.

For a thin fluid disk under translational motion of velocity U, the momentum balance equation can be written as [4]

$$(1 + k)d(VU)/dt = I - F/\rho, \tag{2}$$

where V is disk volume, $k = 1$ is the trailing mass coefficient, and ρ is density. The viscous friction F is primarily determined by friction along the horizontal boundaries of the disk, and we can use the following estimate for this force:

$$F = C_0\nu\rho UD/2, \tag{3}$$

where $C_0 = {}^{32}/_{3}$ [8]. By substituting Eqs. (1), (3) into Eq. (2) and recasting the left side of Eq. (2) to account for the mass balance equation $dV/dt = Q$ as

$$d(UV)/dt = QtdU/dt + QU,$$

we obtain a differential equation for U whose solution takes the form

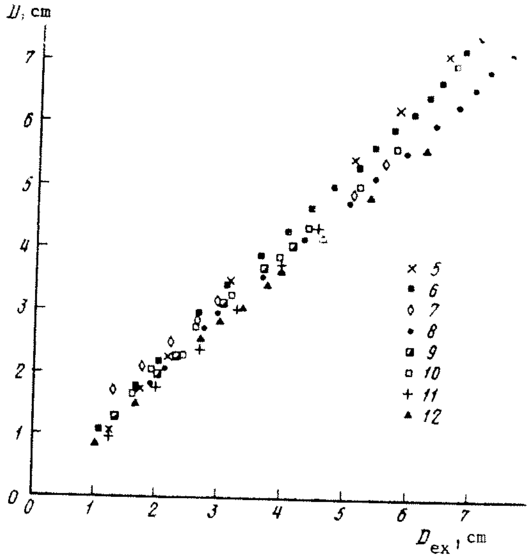
$$U = \frac{2I}{(1 + k)aQt^{1/2}}\left[1 - \frac{1}{at^{1/2}}\left(1 - \exp(-at^{1/2})\right)\right], \tag{4}$$

where $a = \dfrac{C_0C\nu^{9/10}N^{1/5}}{(1 + k)Q^{3/5}}$.

A typical value of a^2 for our experimental conditions is $a^2 \gtrsim 0.5\ s^{-1}$. For $t \gg a^{-2} \approx 2$ s, the current becomes self-similar, and we obtain from Eq. (4) the simple estimate

$$U \approx \frac{2I}{C_0C\nu^{9/10}N^{1/5}Q^{2/5}t^{1/2}}. \tag{5}$$

Estimate (5) for the translational velocity of the spot can be tested experimentally as follows. The velocity U_1 of the leading edge of the spot is



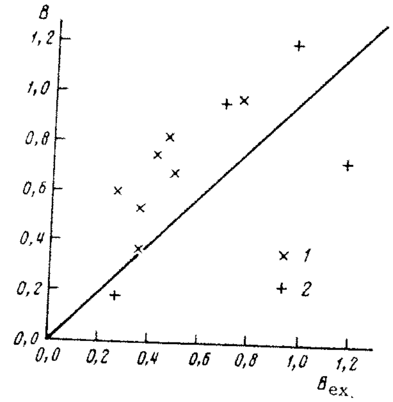


Fig. 3

Fig. 4

Fig. 3. A comparison of spot diameter D estimated from Eq. (1) to the experimentally measured values D_{ex}; $N = 2.1\ s^{-1}$. Points 5-12 correspond to the values of parameters Q and I listed in Table 1.

Fig. 4. A comparison of the calculated B and experimentally measured B_{ex} values of the coefficient B in Eq. (6); N = 2.1 (1), 3.1 s^{-1} (2).

Experiment No.	$Q \cdot 10^{-2}$, cm^2/s	$I \cdot 10^{-3}$, cm^4/s^2	A	Regime
1	0,8	0,2	0,3	Intrusion
2	1,3	0,5	0,5	"
3	2,0	1,2	0,8	"
4	2,3	1,6	1,0	"
5	2,4	1,8	1,1	Dipole
6	3,3	3,4	1,5	"
7	3,7	4,3	1,8	"
8	4,0	5,0	1,9	"
9	4,4	6,0	2,2	"
10	4,7	6,9	2,4	"
11	5,4	9,1	2,9	"
12	6,4	12,7	3,6	"

determined by the sum of the translational velocity of the spot (U) and its uniform expansion (U_0). Using Eqs. (1), (5), we find

$$U_1 = U + U_0 = B/2\sqrt{t}, \qquad (6)$$

where $B = \dfrac{4}{C_0 C\pi} \cdot \dfrac{Q^{8/5}}{\nu^{9/10} N^{1/5} d^2} + \dfrac{C}{2} \cdot \dfrac{N^{1/5} Q^{2/5}}{\nu^{1/10}}$.

The measurement results from U_1 are accurately approximated by a relation of the type

$$U_1 = B_{exp}/2t^n, \qquad (7)$$

where $n \approx 0.48 \pm 0.08$.

A comparison of the calculated (B) and measured (B_{exp}) values of the coefficient demonstrates a satisfactory agreement (Fig. 4).

Using the estimates for U and U_0, we obtain the parameter A as

$$A = U/U_0 = \frac{8I}{C_0 C^2 \nu^{4/5} N^{2/5} Q^{4/5}}. \qquad (8)$$

The table lists the results of 12 experiments conducted for different values of Q (and, consequently, different values of I) as well as values of A calculated from Eq. (8) and lists the observed flow regimes. It is clear from the table that the flow regime transition occurs at $A = A_{\star} \gtrsim 1$.

CONCLUSION

We now attempt to apply our results to interpret field data. We use the well-known example of the mushroom cloud flow [9] resulting from the melting of fast ice in the Tatar Strait. According to [9], the velocity in the jet portion of the flow (of width $d = 2$ km) was $u \gtrsim 10$ $cm \cdot s^{-1}$. We assume the vertical scale of the flow $h \gtrsim 10$ m, a stratification parameter $N = 10^{-2}\ s^{-1}$ (along the fresh water/saline water boundary), and a viscosity $\nu \gtrsim 1\ cm^2 \cdot s^{-1}$ (in this case, this is the vertical momentum exchange coefficient). Estimating I and Q as $I \approx u^2 dh,\ Q \approx udh$, we obtain from Eq. (8) the estimate $A \gtrsim 10^3 >> 1$. The flow regime is clearly a mushroom cloud regime. Note that analogous estimates of the parameter A for the flow of Mediterranean Sea water through the narrow canyons of the Gulf of Cadiz also yield large values of A, which indicates the high probability of the formation of dipole vortices in this region. This was also confirmed by direct measurements during the sixteenth voyage of the Vityaz' scientific research vessel [10]. Preliminary estimates have therefore demonstrated that the mushroom cloud flow regime may be quite widespread not only in the surface layer [9] but also in the column of a stratified ocean. A detailed analysis, however, requires more comprehensive observational data.

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