Stability Theory

Lecture Notes

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and

class 2003:

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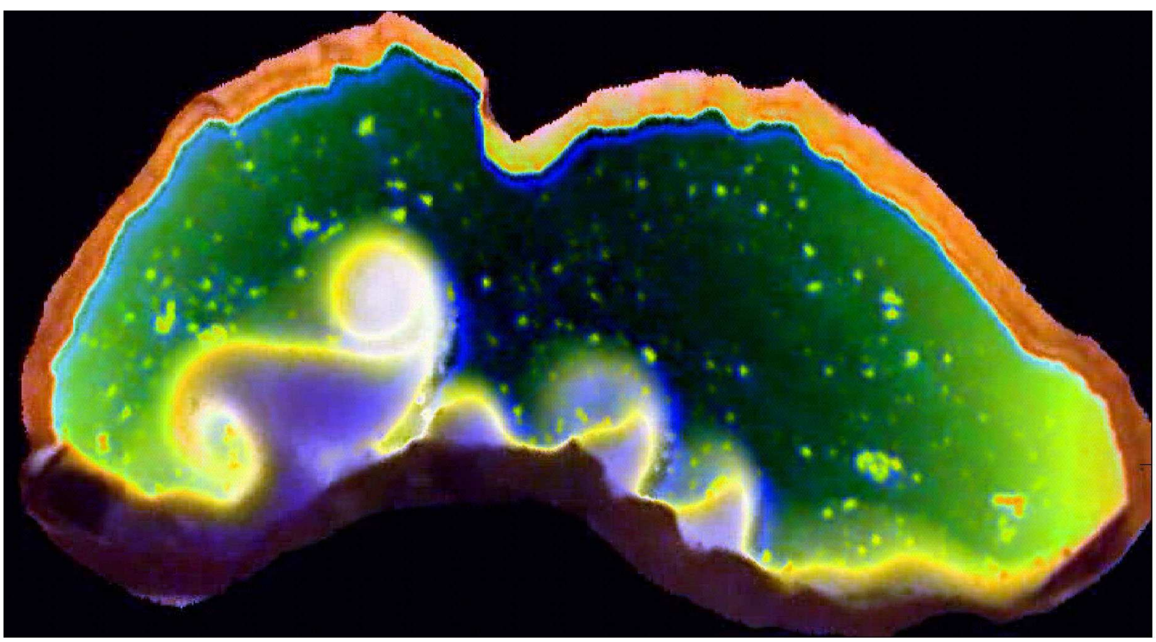

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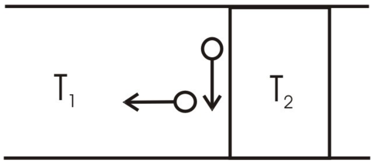


Figure 13: Schematic representation of the formation of diffusive layers in the vicinity of an iceberg.

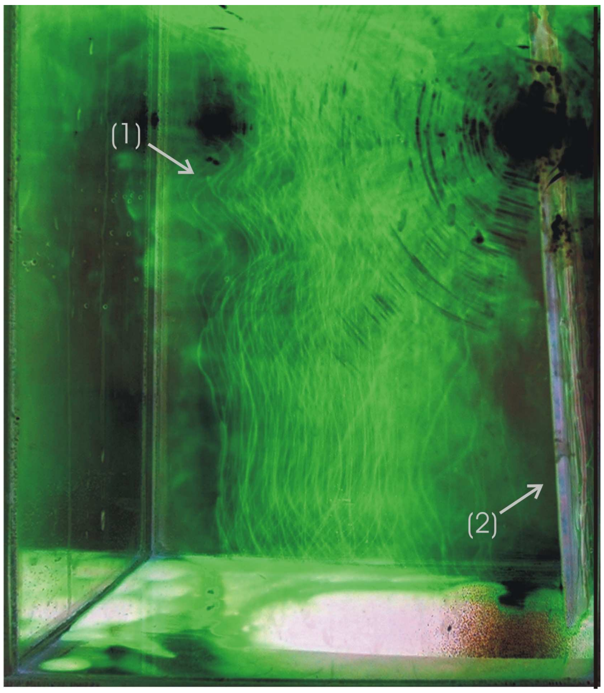


Figure 14: Photograph of demonstration performed in class that shows the formation of diffusive layers in the vicinity of an iceberg. To observe the formation of these layers, potassium permanganate is sprinkled on top of the fluid. Since the potassium permanganate is heavier then the fluid, it sinks, leaving streaks of color. (1) The streaks of color were seen to move away from the sheet of brass indicating the presence of diffusive layers. (2) Sheet of brass that had been cooled using liquid nitrogen to simulate the presence of an iceberg.

Figure 14 clearly shows the horizontal layers (figure 14 (1)) that form in a stable continuously stratified fluid after the addition of a sheet of brass that had been cooled with liquid nitrogen (figure 14 (2)). These layers can extend for long distances away from an iceberg and so we can see that the presence of icebergs can greatly effect oceanic circulation.

References:

[1] Kundu, P. K. and Cohen, I. M., *Fluid Mechanics, 2nd Edition*. Academic Press, San Diego (2002), p.444 - 448

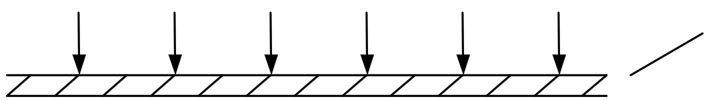
[2] Turner, J. S., *Buoyancy effects in fluids*. Cambridge University Press, Cambridge (1979), Ch. 8

[3] Cushman-Roisin, B., *Introduction to Geophysical Fluid Dynamics.* Prentice-Hall Inc., Englewood Cliffs, New Jersey (1994), p. 36

Lecture 3.

Convection in a deep layer

Benard convection – in thin layers. In oceanography, it is more realistic to consider deep layers of fluid, typical problem—surface cooling in the ocean. It can occur during nightly hours due to combined effect of evaporation and heat radiation, and also in polar regions (deep convection in Labrador or Weddel seas).



This layer with finite buoyancy excess

Deep Ocean

Convection occurs not in the form of cells but rather multiple elements (thermals) which propagate into the deep layer. These elements interact with each other, their numbers decrease as they propagate. They form the mixed layer. This is of course nonlinear flow, but initially the appearance of these cells must be deduced by linear theory.

We can also obtain the estimates for the evolution of the depth of the mixed region which can be important for practical oceanographic application. Here we do not solve any equations but rather use the dimensional analysis.

Example of a thin (heavy) layer with buoyancy excess,

$$B = f\,\frac{\Delta\rho}{\rho_0} h_0\,,$$

H || B, t, v => Π-theorem gives 2 parameters of independent dimensions

[L], $\frac{L^2}{T^2}, T, \frac{L^2}{T}$ 3-2=1 non-dimension parameter

$$\frac{H}{B^{1/2}t} = f(R_e)\,,\ R_e = \frac{Bt}{v}$$ ----analog of the Reynolds number.

where f is some non-dimensional function of non-dimensional argument R_e (R_e>>1 in practical cases)

Assume a complete similarity of f(R_e) with respect to its argument when $R_e \to \infty$:

$$f(\mathrm{Re} >> 1) = \mathrm{co}n\mathrm{s}t = c\ ,$$

$$=> H = ctB^{1/2}\ .$$

c cannot be obtained from dimensional analysis. Experiments give $c \approx 0.4$.

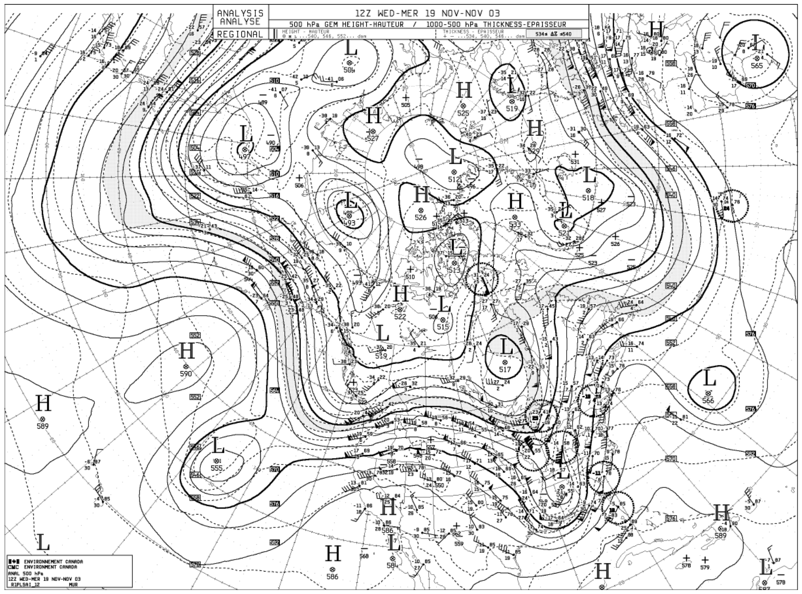


Figure 14: Baroclinic instabilities in the Arctic Polar Front as observed on Nov. 19th 2003

Figure 15: The development of baroclinic instability as shown in the lab demonstration.