Experiments on the evolution of gravitational instability of an overturned, initially stably stratified fluid

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Unstable density stratifications were created in the laboratory by rapid overturning of a narrow tank containing an initially stable density structure. The experiments were carried out for two different initial density distributions: (i) a two-layer (steplike) and (ii) a linear stratification. For the former case the depth of the mixing layer was found to increase linearly with time. The number of convective elements (thermal-like flow structures) present at the front of the mixing layer was observed to decrease with time, through the mechanism of subsequent pairing. In the case of an initially linear stratification the flow evolution is characterized by a number of distinct stages: different modes of instability emerge subsequently through the entire fluid column, leading to the formation of horizontal layers, which finally break up into thermal-like convective flow structures.

I. INTRODUCTION

A system consisting of heavier fluid lying on top of less dense fluid is gravitationally unstable, and will generally give rise to overturning motions in the fluid. This overturning will continue until an ultimately stable stratification is established. A typical geophysical example of this instability process can be found in the ocean, in cases of surface cooling. This situation occurs during nightly hours, when the cooling is caused by the combined effect of evaporation and heat radiation, and also in polar regions. It appears that the process of transition from unstable to stable stratification strongly depends on the initial density distribution.[1] For the case of a constant buoyancy flux at the surface it was found both theoretically[2] and in laboratory experiments[3] that the depth (h) of the mixing layer increases with time as $h\sim t^{3/2}$. On the other hand, when the buoyancy excess is initially concentrated in a thin fluid layer at the surface, with the density anomaly thus taking the appearance of a Dirac delta function in the vertical coordinate, a different asymptotic behavior is observed:[4] $h\sim t$. This particular case of impulsive cooling was also considered theoretically[5,6] within the framework of the semi-empirical turbulence theory. The analysis gives the depth of the mixing layer in the asymptotic form: $h=cB^{1/2}t$, where B is the excess of buoyancy and c some proportionality constant. The results of two laboratory tests[4] and a two-dimensional numerical simulation[7] of the mentioned experiment suggest that the c value lies in the range 0.25–0.6. Knowledge of the exact value of the coefficient c is important for many geophysical applications, in particular for prediction of the depth of the turbulent upper ocean layer. Thus, more accurate experimental estimates for c in a broad range of external parameters are desirable.

Another extreme situation is met in the case of two fluid layers of equal depths and with contrasting densities; in that situation the mixing-layer depth is observed[8,9] to grow as $h\sim t^2$.

Most previous studies of gravitational instability concerned the behavior of an overturned two-layer fluid system. In the present study the attention is also focused on the gravitational instability in an overturned continuously stratified fluid. This problem is relevant to a number of geophysical situations, such as the unstable density distributions locally generated by breaking internal waves in a continuously stratified ocean. The laboratory experiments described in this paper reveal that in the case of an overturned linearly stratified fluid column thin horizontal layers emerge during the adjustment process. This formation of layers may possibly be connected with the phenomenon of steplike fine structure as observed in a continuously stratified ocean.

II. EXPERIMENTAL ARRANGEMENT

Here we present the results of an experimental study of the evolution of gravitational instability for two different initial density distributions, viz., a steplike and a continuous stratification. In the first case a density step was created with an initial height (h_0) being much less than the total height (H) of the tank ($h_0 \ll H$). In the second case a linear density distribution was used.

The experiments were conducted in a narrow transparent tank of dimensions $98\times55\times2$ cm, which could be rotated about its long, horizontal axis. The density stratification of the fluid was created by variation in the salt concentration in the vertical direction and was measured by a conductivity probe.[10] A linear density stratification was established by application of the standard "two-bucket" technique.[11] Weak stratifications were produced by filling very slowly, typically during 1–2 h, and a small porous sphere at the end of the filling tube was used to avoid any mixing during the filling process. For visualiza-

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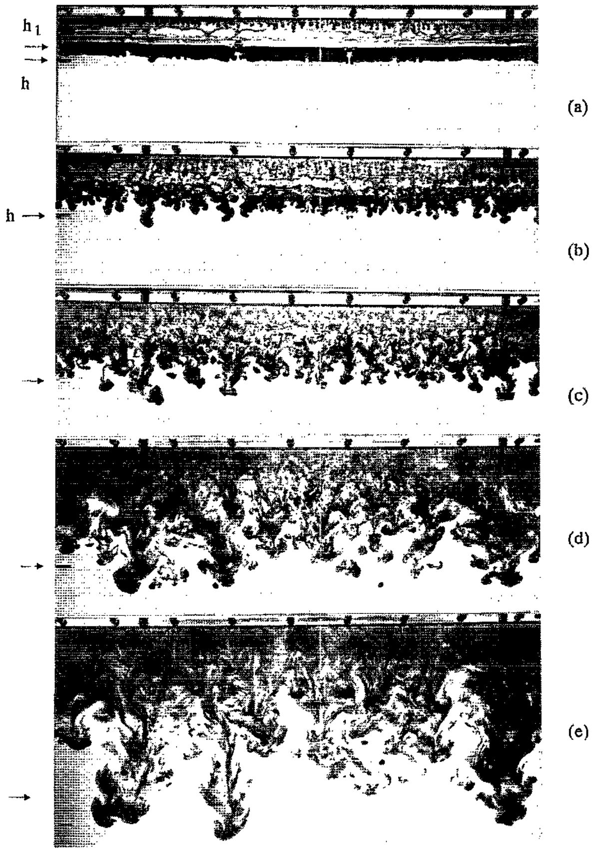

FIG. 1. Sequence of photographs illustrating the evolution of instability when a thin ($h_0=1$ cm) layer of heavy dark-colored fluid lies on top of a layer of lighter fluid ($\Delta\rho=1.6\times10^{-4}$ g cm^{-3}) after the the overturning of the tank. The photographs are taken at (a) $t=6$ sec, (b) 11 sec, (c) 17 sec, (d) 34 sec, (e) 55 sec, after the overturning. The distance between the bolts (black dots) in the upper wall of the tank is 10 cm. The arrows indicate the initial position (h_1) of the heavy layer and the estimated mean position (h) of the leading fronts of the convective elements at subsequent times.

tion of the flow we did not use a dye (which would cause changes in the fluid density) but the pH indicator thymol blue.[12] For some experiments a linearly varying dye concentration was created simultaneously with the density stratification, so that the darker color corresponds with the saltier fluid. An unstable density stratification was obtained by turning the tank upside down, a method also used by Dikarev and Zatsepin.[4] The time of this half-revolution was about 2 sec. The main measurements were made photographically, and the conductivity probe was used to measure the initial and final density distributions.

III. OBSERVATIONS

A. Steplike stratification

In all the experiments with a two-layer stratification the initial height of the layer of denser fluid was $h_0=1$ cm. The density difference ($\Delta\rho$) between the layers was varied in the range $\Delta\rho=(0.2\text{–}16)10^{-3}$ g cm^{-3}. The sequence of photographs in Fig. 1 illustrates the typical evolution of the flow. Since the fluid in the tank was at rest before the overturning, it is clearly impossible for the vorticity associated with the solid-body rotation to be imparted to the interior of the fluid during the short period of the overturning. Thus, the fluid does not follow the rotation of the tank but it only undergoes an irrotational displacement with respect to the tank. As a result, this displacement rotates the interface between layers, or the isopycnals in the case of a linear stratification, respectively. This process was studied experimentally by Simpson and Linden.[13] According to the results of that study, inverting the tank by rotation would cause the interface to tilt at an angle $\theta\simeq\tan^{-1}2\pi$; for the present configuration this implies that the layer will tilt down over a vertical distance of approximately 5 cm. This estimate does not take into account the creation of thin diffusive boundary layers (vortex sheets) at the walls during the overturning. One can estimate the thickness of the vortex sheets as $\delta\simeq\sqrt{\nu t_0}\simeq0.15$ cm, $t_0\simeq2$ sec being the duration of the overturning. The residuals of such a sheet are clearly seen in the upper part of the tank in Fig. 1(a). Thus, the position of the thin heavy layer after the overturning of the tank is clarified: it is localized near the upper part of the back wall at depth $h_1\simeq4\text{–}5$ cm [the direction of rotation is indicated in Fig. 4(a), see below]. During the rapid overturning the inertia effects prevail and the buoyancy force does not play a significant role. After the overturning, however, the buoyancy force begins to generate subsequent motions within the fluid. A longitudinal horizontal perturbation mode develops and small convective (thermal-like) elements form along the layer [Fig. 1(b)]. These convective elements are localized jetlike flow structures with a typical mushroom shape.

As time progresses, the convective elements grow and become large compared to the width of the tank, implying that the flow at this stage can be considered as quasi-two dimensional. In this stage of the flow evolution the potential energy of the system decreases as a result of downward movement of heavier convective elements. Note, that it is not essential whether the flow is purely two dimensional or partly three dimensional. The above-mentioned asymptotic dependence $h\sim Bt^{1/2}$ must be the same for both cases. However, the question arises: Does the viscous friction at the vertical walls of the tank significantly influence the flow dynamics? The typical duration of the experiments is less than 10^2 sec—this is the maximum time needed for the convective structures to reach the bottom of the tank. A simple estimate for the maximum thickness of the diffusive boundary layers gives: $\delta\approx\sqrt{\nu t}\leqslant1$ cm, which is comparable with the width of the tank. However, the measurements (to be discussed below, see Fig. 2) indicate a good correspondence with the theoretically predicted intermediate-asymptotic law of the flow evolution. This observation implies that the tank is wide enough, and the vertical walls do not essentially influence the global characteristics of the flow.

Neglecting the details of the flow evolution in the initial period, consider the depth (h) of the convective region advancing through the fluid at this intermediate-asymptotic stage ($h_0\ll h<H$). Here h is measured from the initial position of the heavy layer at level h_1 [see Fig. 1(a)].

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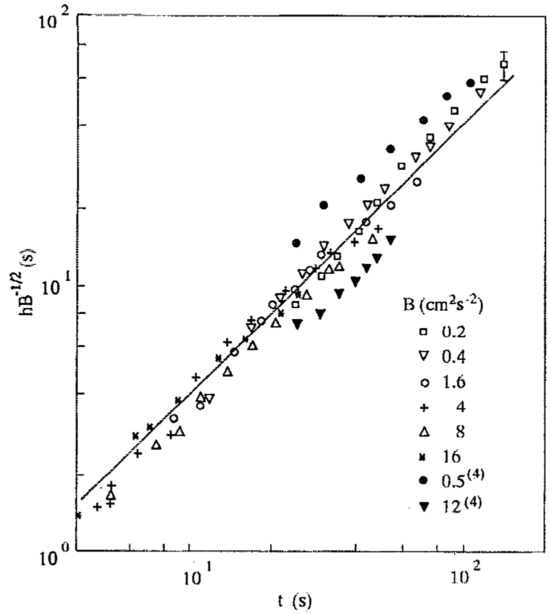

FIG. 2. Plot of the normalized width $h/B^{1/2}$ of the convective region versus the time t measured from the beginning of the experiment. The data obtained in Ref. 4 are shown by (●) and (▼).

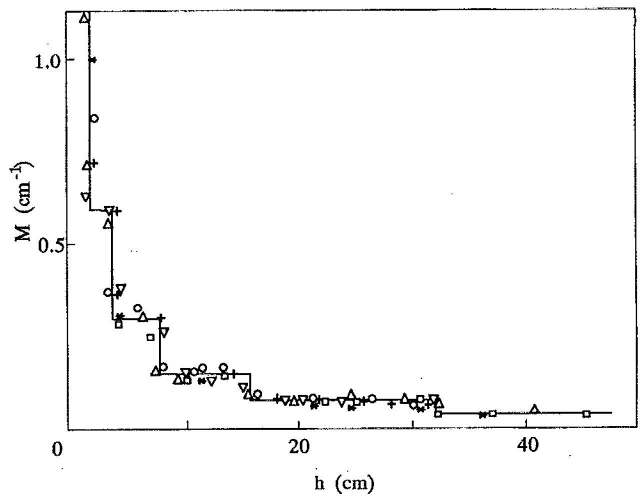

FIG. 3. Plot of the number M of convective elements per unit length versus the depth h of the convective region. The steplike solid line represents the theoretical prediction.

The lower boundary of the convective region was determined by eye as the mean position of the leading fronts of the convective elements. Although the scatter in the leading positions of the convective elements is significant at any stage of the flow evolution (see Fig. 1), the error in the determination of h is much less: because of the averaging it is inversely proportional to the square root of the number of elements. The magnitude of the typical error is shown by the bars in Fig. 2. The parameters governing the flow dynamics are the buoyancy excess of the heavy layer, $B=g(\Delta\rho/\rho_0)h_0$, the time t, measured from the beginning of the experiment, and most likely the kinematic viscosity, ν. Dimensional analysis then gives

$$h/(B^{1/2}t)=f(\mathrm{Re}), \tag{1}$$

where f is some nondimensional function of the nondimensional argument $\mathrm{Re}=Bt/\nu$ which can be considered as the analog of the Reynolds number of the flow. In our experiments $\mathrm{Re}\gg 1$ for $t\gg t_0$. As a first step in the prediction of the asymptotic behavior of the function $f(\mathrm{Re})$, assume a complete similarity of $f(\mathrm{Re})$ with respect to the parameter Re: $f(\mathrm{Re}\gg 1)=\mathrm{const}=c$. Experimentally obtained values of $h/B^{1/2}$ over the range of $B=(0.2\text{–}16)\ \mathrm{cm^2\,sec^{-2}}$ are shown graphically in Fig. 2. For comparison, all data obtained by Dikarev and Zatsepin[4] are also plotted on the graph. It appears that during the considered intermediate-asymptotic stage the depth of the convective region increases linearly with time according to

$$h=ctB^{1/2}, \tag{2}$$

where the constant c has a value 0.38 ± 0.02 (note that only the results obtained in the present experiments were used to determine c).

It can be observed from Fig. 1 that the number of convective elements per unit length in the horizontal direction decreases with time. Their number (M) in the vicinity of the front of the mixing region was estimated from the photographs. The data points in Fig. 3 show the measured values of M versus the mean depth (h) of the convective region. As a first approximation, the experimental data of M can be described by a smooth curve of the form $M=\alpha^{-1}h^{-1}$, where $\alpha=\mathrm{const}\approx0.55$. [This coefficient α is in fact the entrainment coefficient, and for isolated thermals it has the value $\alpha=0.45$ (Ref. 14).] One can thus obtain an estimate for the mean horizontal length scale (L) of the convective elements: $L\approx M^{-1}\approx\alpha h$. Hence L increases linearly with h.

This simple approximation, however, does not consider the details of the process which leads to changes of the number of convective elements with time. A closer inspection of the experimental data reveals that the relationship $M(h)$ is not smooth but rather has a steplike structure, as was pointed out by Dikarev and Zatsepin.[4] Note, that a similar steplike behavior of the function $M^{-1}(h)$ can be clearly observed in Wooding's[15] experimental data (see Fig. 3 in that paper) on the growth of convective fingers in a Hele–Shaw cell. Now, on the basis of experience with vortex dipole interactions,[16] we can propose a model to explain that experimental fact. The basic assumption in the model is that at the intermediate-asymptotic stage the structure of the convective layer is self-similar. The nature of this self-similarity is clear. Owing to the entrainment the size of the moving convective elements increases continually. At some distance h_i the horizontal length scale L_i of the elements becomes equal to the distance between adjacent elements $L_i=M_i^{-1}$. As a consequence, the elements begin to interact. During this process relatively weaker elements decelerate and get entrained into the tail of the stronger ones. Thus the pairing of the structures decreases their number approximately by

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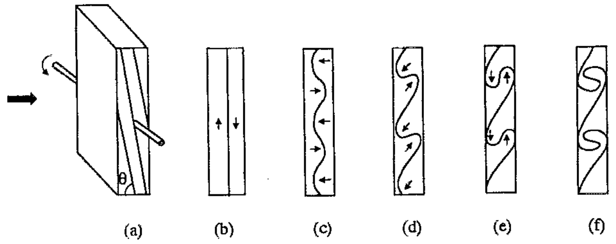

FIG. 4. Sketch of the inclined isopycnals (a) and the motions in subsequent periods of the flow evolution for the case of an initially linear stratification. The big arrow indicates the direction at which the photographs were taken; the small arrow indicates the direction of rotation. (b) The global mode of disturbances—upward and downward motion in vertical layers; (c), (d), and (e) the subsequent horizontal layering and overturning of alternating heavy and light layers; (f) the appearance of fine structure at the boundaries between the horizontal layers.

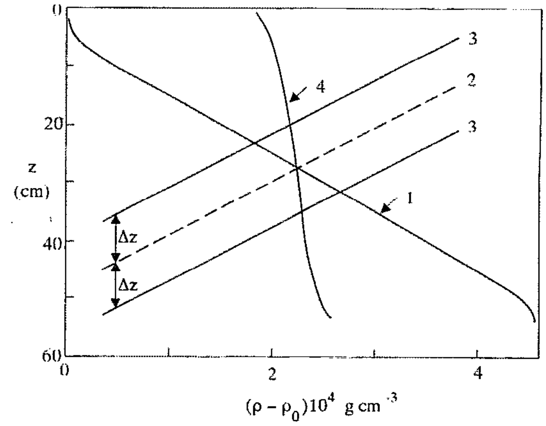

FIG. 5. Typical vertical density profiles measured before the overturning of the tank (1) and at the end of the experiment (4). The profiles after overturning in the vertical midplane of the tank (2) and at the front and back face walls (3) are also shown schematically.

a factor of 2, so that $M_{i+1}=\frac{1}{2}M_i$ below the level h_i where the pairing occurs. A number of such pairings takes place as the convective elements progress. The subsequent levels h_i at which the number of elements decreases by a factor of 2 can be easily estimated assuming that the entrainment coefficient α is a constant: $\alpha(h_i+h_{i+1})/2=M_i^{-1}$ and $h_{i+1}=2h_i$. This steplike function is indicated in Fig. 3 by the solid line.

B. Linear stratification

The second qualitative series of experiments was carried out in order to study the evolution of instability of a fluid layer with an approximately constant undisturbed density gradient $d\rho/dz=\gamma=\text{const}$, with the z axis directed downward, and $\gamma<0$. In order to be able to conveniently observe the flow behavior a very weak initial density gradient was applied, $\gamma\approx(10^{-5}\text{–}10^{-4})\text{g cm}^{-4}$, such that the time scale of the flow evolution was stretched to typically 10–100 sec. As was mentioned above, it is not possible to produce a horizontally homogeneous unstable linear stratification by overturning the tank. Due to the irrotational displacement with respect to the tank, the isopycnals rotate at an angle $\theta=\tan^{-1}2\pi$ [Fig. 4(a)]. The resulting density distribution in the tank after overturning will be approximately the following, see Fig. 5. At the midplane, parallel to the front and back walls of the tank, the density profile (indicated by 2) is inversed compared to the initial profile (1) before the overturning of the tank. At the front and back walls the density distribution is represented by the profiles (3) with the same unstable density gradient, but shifted up and down, respectively, with respect to the midplane profile (2), over a distance $\Delta z=(l/2)\tan\theta=\pi l$, l being the width of the tank. Thus the fluid at the front wall of the tank turns out to be lighter than the fluid at the back wall, so that immediately after the overturning of the tank the linearly stratified fluid begins to move upward and downward along the front and back walls of the tank in a uniform fashion, as shown schematically in Fig. 4(b). As a result, the fluid separates into two vertical layers in which the density at any horizontal level differs by the quantity $\Delta\rho\approx2\gamma[h(t)+\Delta z]$, where $h(t)$ is a height over which one layer shifts up and the other one shifts down from their initial position after the overturning [Fig. 4(b)]. Redistribution of the density leads to changes in the hydrostatic pressure. The pressure difference between two sliding layers is in good approximation given by $\Delta p\approx\Delta\rho gz$ (the origin $z=0$ is taken at the half-depth of the tank). The pressure difference has its maximum value at the bottom ($z=H/2$), and the same negative value at the top ($z=-H/2$). Thus the vertical interface between two layers becomes unstable and the layers begin to intrude into one another forming horizontal layers of different density [Fig. 4(c)].

On the photographs shown in Figs. 6(b) and 6(c) one can observe the appearance of these horizontal layers of relatively dark and light fluid with approximately equal vertical length scale (of about 6–8 cm). Observations demonstrate that initially the layers at the top and at the bottom show up with a stronger contrast than in the center of the tank. This is a consequence of the actual distribution of the pressure difference. Thus, the system turns out to consist of alternating horizontal layers of relatively denser and lighter fluid. In this system one can expect that the adjacent layers of heavy and light fluid begin to overturn as do the layers in the case of a sinusoidal vertical density distribution, as considered by Batchelor and Nitsche.[17] Observations reveal that each pair of layers, consisting of more dense fluid lying above less dense fluid, subsequently begins to overturn. The denser fluid begins to flow down along the front wall of the tank and the lighter fluid in turn flows up along the opposite wall. The mean vertical motions here are of opposite directions compared to the first phase when the fluid moved up along the front wall and down at the back wall [compare Figs. 4(b) and 4(e)]. This motion causes the detachment of layers from the walls [Fig. 4(f)] which in turn leads to the development of a fine structure at the boundaries between the layers [Fig. 6(c)]. It appears that the number of intermediate bifurcations, occurring when the layers of heavier fluid gradually move down oscillating from one wall to another and the lighter fluid moves up in the opposite way, depends on the initial strat-

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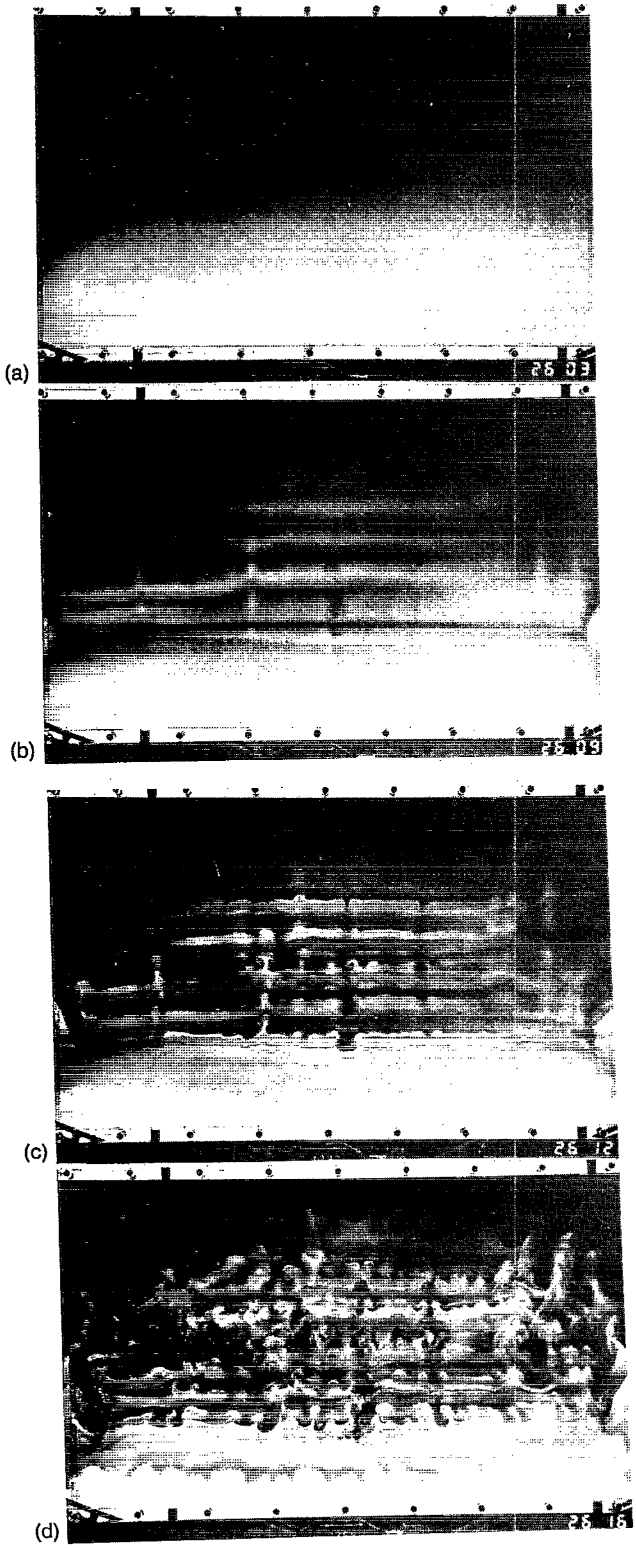

FIG. 6. Sequence of photographs showing the evolution of the instability for the case of an initially linear stratification. The initial density gradient (see Fig. 5) was $\gamma \approx 10^{-5}$ g cm^{-4}. The photographs were taken at (a) $t=22$ sec, (b) 43 sec, (c) 49 sec, (d) 55 sec, (e) 62 sec, (f) 76 sec, measured after the start of the experiment.

ification of the fluid. During this motion small-scale disturbances develop in the longitudinal direction at the boundaries of the layers [Fig. 6(d)]. Numerous convective elements begin to grow coherently through the whole body of fluid, at this stage consisting of alternating unstable horizontal layers [Fig. 6(e)]. The subsequent development of convective flow structures leads to an effective turbulent mixing of fluid [Fig. 6(f)], as can also be clearly observed from the comparison of the initial (1) (before the overturning) and final (4) vertical density distributions as shown in Fig. 5. The evolution of the instability of the overturned layer of a linearly stratified fluid can thus be considered as a sequence of intermediate bifurcations leading to the gradual decrease of the vertical scale of motion (Δh), and the decrease of the potential energy of the system.

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It is useful to define the Rayleigh number of the flow as $\mathrm{Ra}=\Delta h^4\gamma g/(\rho\nu\kappa)$, with κ being the molecular diffusivity of salt. Initially the Rayleigh number is extremely large, with values up to 10^{12}–10^{13} for $\Delta h=H$, but as a result of the subsequent bifurcations it decreases. Visual observations from photographs [see also Fig. 6(e)] have revealed that the minimal vertical length scale of the horizontal layers is approximately $\Delta h^*\approx 1$ cm. This gives the estimate: $\mathrm{Ra}^*\approx 10^5$, which is still 150 times larger than the classical critical value $\mathrm{Ra}_c=27\pi^4/4$, derived by Rayleigh as the criterion of the beginning of instability between free horizontal boundaries. This explains the fact that the horizontal layers are still unstable and break up into thermal-like structures, which further decrease the length scale of the density inhomogeneities. For more realistic estimates detailed data on the density field during the evolution of instability are needed. Because of technical difficulties, however, we have not been able to perform these measurements.

IV. CONCLUSIONS

The relatively simple experiments described in this paper demonstrate that the gravitational instability of a stratified fluid may give rise to rather complex phenomena. In the case of an initial *two-layer* stratification it was observed that the depth of the turbulent mixing layer that arises after the start of the experiment increases to very good approximation linearly in time. The front of this deepening mixing layer is characterized by the occurrence of thermal-like flow structures, whose horizontal scales increase in time, through the mechanism of subsequent pairing. In the case of an unstable *linear* density stratification one observes the rapid formation of horizontal layers of alternating heavier and lighter fluid. The vertical dimension of these layers is approximately equal over the full depth of the tank. Since this new stratification is again unstable, the layers soon break up into small thermal-like flow structures in a similar fashion as in the two-layer case, but now distributed over the full depth of the fluid. The complicated interaction between these flow structures gives the flow a very irregular appearance, and results in an effective turbulent mixing of fluid and therewith in a rapid decrease of the potential energy of the system.

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