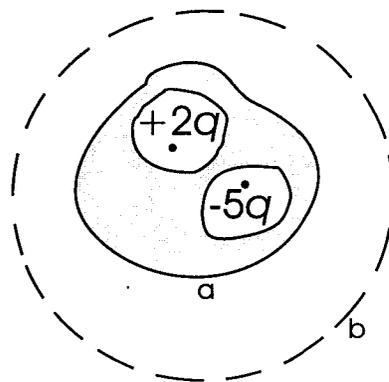


SECTION 1: MULTIPLE CHOICE. (answer in the box provided for each question)

- [2] 1. An **UNCHARGED** conducting object *a* contains two cavities. One cavity contains a point charge of $+2q$ and the other contains a point charge of $-5q$. Which of the following statements **IS TRUE**?

B

- A. The total electric flux through a spherical surface *b* completely enclosing the object is $\frac{+3q}{\epsilon_0}$.
- B. The total electric flux through a spherical surface *b* completely enclosing the object is $\frac{-3q}{\epsilon_0}$.
- C. The total electric flux through a spherical surface *b* completely enclosing the object is 0.
- D. The total electric flux through a spherical surface *b* completely enclosing the object depends on the area of the surface.



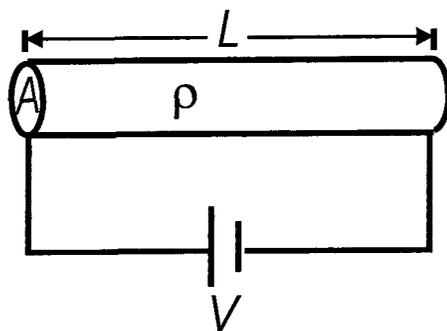
- [2] 2. Which of the following statements **IS TRUE**?

D

- A. The electric potential on the surface of a charged conductor is highest at the point where the surface is most sharp.
- B. The electric field at the surface of a charged conductor is parallel to the surface of the conductor.
- C. The excess charge on a charged conductor is uniformly distributed over its volume.
- D. The electric field just outside the surface of a charged conductor depends on the surface charge density.

- [2] 3. A cylinder with a cross-sectional area *A* has a length *L*. The resistivity of the cylinder material is ρ . A battery is used to apply a potential difference ΔV across the length of the cylinder. Which of the following statements **IS TRUE**?

C

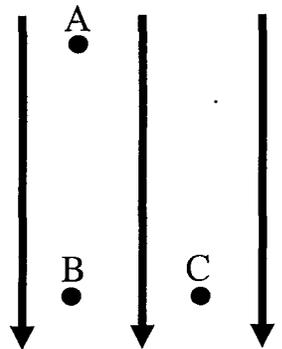


- A. $V = \frac{IA}{\rho L}$
- B. $V = \frac{I\rho A}{L}$
- C. $V = \frac{I\rho L}{A}$
- D. $V = \frac{\rho L}{IA}$
- E. $V = \frac{I\rho}{AL}$

[2] 4. Three points are located in a region of uniform electric field as shown. The line from A→B is parallel to the field and the line from B→C is perpendicular to the field. Which statement **IS TRUE**?

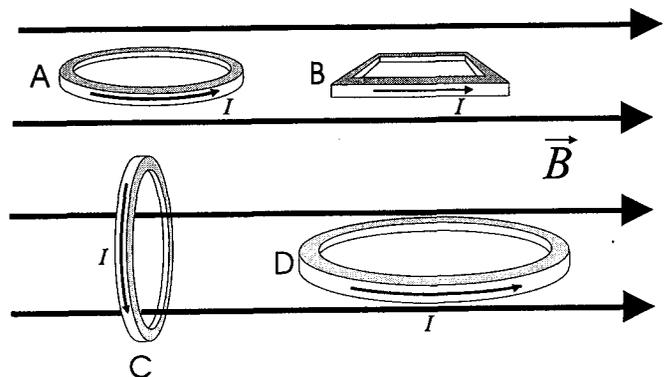
A

- A. The potential difference $V_B - V_A$ is negative.
- B. The potential difference $V_B - V_A$ is greater than the potential difference $V_C - V_A$.
- C. The potential difference $V_B - V_A$ is less than the potential difference $V_C - V_A$.
- D. The electric potentials at points A, B, and C are equal.
- E. The potential $V_B - V_A$ is zero.



[2] 5. Four loops, each carrying a circulating current I are located in a region of uniform magnetic field as shown.

- Loop A is a circle with an area of 0.02 m^2 and the field is parallel to the plane of the loop.
- Loop B is a square with an area of 0.02 m^2 and the field is parallel to the plane of the loop.
- Loop C is a circle with an area of 0.02 m^2 and the field is perpendicular to the plane of the loop.
- Loop D is a circle with an area of 0.04 m^2 and the field is parallel to the plane of the loop.



Which of the following statements **IS TRUE**?

D

- A. The torque on loop B is larger than the torque on loop A.
- B. The torque on loop C is larger than the torque on loop A.
- C. The torque on loop C is equal to the torque on loop A.
- D. The torque on loop D is larger than the torque on loop A.
- E. The torque on loop D is equal to the torque on loop A.

SECTION 2: DO ALL FOUR (4) QUESTIONS

[10] 6. A thin insulating rod is bent into a half circle of radius $r = 0.05 \text{ m}$ and is uniformly charged with a charge per unit length of $12 \mu\text{C}/\text{m}$. Relative to the centre of the half circle, a point charge $q = -5.0 \mu\text{C}$ is located at 0.1 m in the x direction and 0.05 m in the y direction as shown.

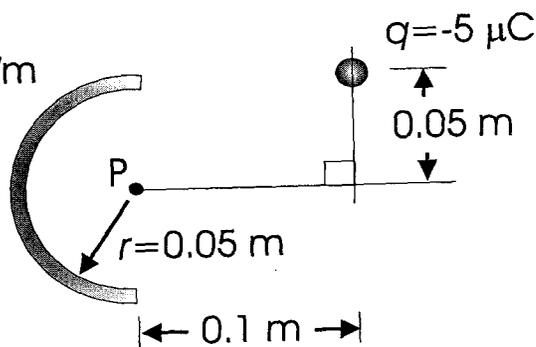
(a) What is the electric potential at a point P located at the centre of the circle, as shown, if electric potential is taken to be 0 at an infinite distance from point P ?

(b) How much work would be done in moving a proton from infinity to point P ?

(a) FOR THE POINT CHARGE

$$V_{pt} = \frac{kq}{r} = \frac{(8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2})(-5 \times 10^{-6} \text{ C})}{\sqrt{(0.1 \text{ m})^2 + (0.05 \text{ m})^2}} \quad \lambda = 12 \mu\text{C}/\text{m}$$

$$V_{pt} = -4.02 \times 10^5 \text{ V}$$



FOR THE BENT ROD

DISTANCE FROM ROD TO POINT P IS ALWAYS r , SO

$$V_{rod} = \int dV = \int \frac{k dq}{r} = \int \frac{k \lambda ds}{r} = \frac{k \lambda}{r} \int_0^{2\pi r} ds = \frac{k \lambda 2\pi r}{r} = k \lambda \pi$$

$$V_{rod} = (8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2})(12 \times 10^{-6} \text{ C}/\text{m})\pi$$

$$V_{rod} = 3.39 \times 10^5 \text{ V}$$

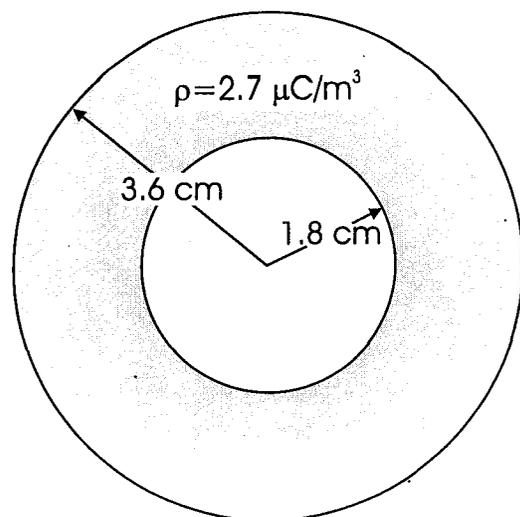
TOTAL

$$V_{TOTAL} = V_{pt} + V_{rod} = -4.02 \times 10^5 \text{ V} + 3.39 \times 10^5 \text{ V} = -6.31 \times 10^4 \text{ V}$$

(b) $W = +q \Delta V = +q (V_B - V_A) = + (1.6 \times 10^{-19} \text{ C})(-6.31 \times 10^4 \text{ V} - 0 \text{ V})$
 $W = -1.01 \times 10^{-14} \text{ J}$ DONE BY AN EXTERNAL AGENT (MOVING A POSITIVELY CHARGED OBJECT TO A REGION OF MORE NEGATIVE POTENTIAL)

[10] 7. An insulating shell consists of a sphere with a radius of 3.6 cm and a concentric spherical cavity with a radius of 1.8 cm as shown. This shell is uniformly charged with a density of $2.7 \mu\text{C}/\text{m}^3$.

- (a) What is the total charge on this object?
 (b) What is the magnitude of the electric field 1.0 cm from the centre of the shell?
 (c) What is the magnitude of the electric field 5.0 cm from the centre of the shell?
 (d) What is the magnitude of the electric field 2.5 cm from the centre of the shell?



$$\textcircled{a} \quad Q = \rho V = \rho \left(\frac{4}{3} \pi (r_{\text{outer}}^3 - r_{\text{inner}}^3) \right)$$

$$Q = (2.7 \times 10^{-6} \text{ C}/\text{m}^3) \frac{4}{3} \pi \left[(0.036 \text{ m})^3 - (0.018 \text{ m})^3 \right]$$

$$Q = (2.7 \times 10^{-6} \text{ C}/\text{m}^3) (1.710 \times 10^{-4} \text{ m}^3) = 4.62 \times 10^{-10} \text{ C} = 4.62 \times 10^{-4} \mu\text{C}$$

$$\textcircled{b} \quad \Phi = \oint \vec{E} \cdot d\vec{A} = EA = \frac{q_{\text{enclosed}}}{\epsilon_0} = 0 \Rightarrow |\vec{E}|_{1.0 \text{ cm}} = 0 \text{ V/m}$$

$$\textcircled{c} \quad E = \frac{q_{\text{enclosed}}}{\epsilon_0 A} = \frac{4.62 \times 10^{-4} \mu\text{C}}{(8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}) [4\pi (0.05 \text{ m})^2]} = 1660 \text{ V/m}$$

$$\textcircled{d} \quad q_{\text{enclosed}} = \rho V = \rho \left[\frac{4}{3} \pi (r_{\text{outer}}^3 - r_{\text{inner}}^3) \right]$$

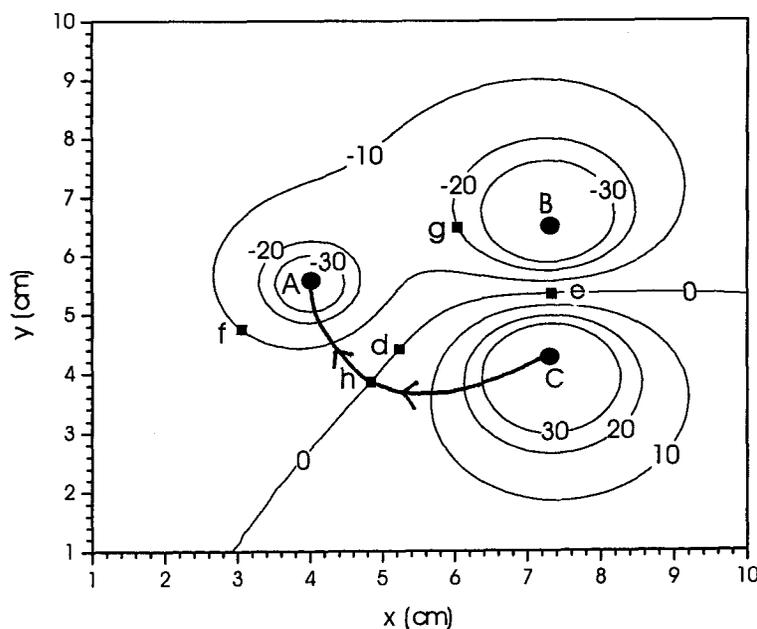
$$= (2.7 \times 10^{-6} \text{ C}/\text{m}^3) \frac{4}{3} \pi \left[(0.025 \text{ m})^3 - (0.018 \text{ m})^3 \right]$$

$$= (2.7 \times 10^{-6} \text{ C}/\text{m}^3) (4.10 \times 10^{-5} \text{ m}^3)$$

$$= 1.11 \times 10^{-10} \text{ C}$$

$$\vec{E} = \frac{q_{\text{enclosed}}}{\epsilon_0 A} = \frac{1.11 \times 10^{-10} \text{ C}}{(8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}) [4\pi (0.025 \text{ m})^2]} = 1590 \text{ V/m}$$

- [10] 8. The graph below shows equipotential lines around three point charges; **A**, **B**, and **C**. The lines are drawn in steps of 10 V for all voltages between -30 V and +30 V. The electric potential is taken to be 0 at infinity.



- (a) Which two charges have the same sign? What is that sign? **A & B ARE NEGATIVE**
- (b) Is the magnitude of the electric field larger at point **d** or at point **e**? Briefly explain your choice.
- (c) Using the scale on the **y**-axis and the equipotentials, estimate the magnitude of the electric field at point **e**.
- (d) Carefully draw the electric field line that starts on one charge, passes through point **h**, and ends on another charge. Be sure that the direction of the field line and the charges on which it starts and ends are clearly shown.
- (e) How much work must an external agent do in order to move an electron from point **f** to point **g**? (hint: be careful with the sign)

(b) $|\vec{E}| = \frac{dV}{dr}$, so a larger distance between equipotential lines means a smaller field. Thus, the field is larger at point e.

(c) $|\vec{E}| = E_y = \frac{dV_y}{dy} = \frac{20 \text{ V}}{0.4 \text{ cm}} = 50 \text{ V/cm} = 5,000 \text{ V/m}$

(d) SEE DIAGRAM ABOVE.
 • LINE STARTS AT C AND ENDS AT A
 • LINE IS PERPENDICULAR TO EACH EQUIPOTENTIAL LINE IT CROSSES.

(e) $W = q\Delta V = (-1.6 \times 10^{-19} \text{ C}) \underbrace{(-20 \text{ V} - (-10 \text{ V}))}_{-10 \text{ V}} = 1.6 \times 10^{-18} \text{ J}$
 REQUIRES POSITIVE WORK FOR AN EXTERNAL AGENT TO MOVE A NEGATIVELY CHARGED OBJECT TO A REGION OF MORE NEGATIVE POTENTIAL.

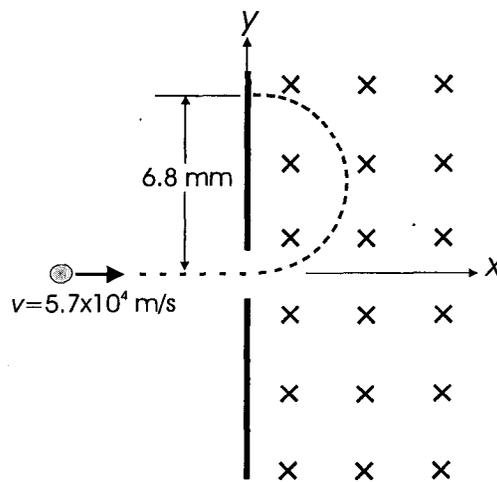
[10] 9. A uniform magnetic field, directed into the page as shown, exists to the right of a barrier. An ion with a mass of 2.5×10^{-26} kg and a speed of 5.7×10^4 m/s enters this region of uniform magnetic field through an aperture at the origin as shown. The ion moves in a counterclockwise direction around a semicircular path that intersects the barrier 6.8 mm above the aperture as shown.

(a) Is the ion positively charged or negatively charged?

(b) What is the centripetal acceleration of the ion when it is in the region of uniform magnetic field?

(c) Assuming that the **magnitude** of the charge on the ion is e (1.6×10^{-19} C), what is the magnitude of the magnetic field?

(d) A negative ion having the same speed and magnitude of charge but a mass of 1.75×10^{-26} kg is projected through the aperture and into the region of magnetic field. Where will its path intersect the barrier?



$$(b) \frac{v^2}{r} = a = \frac{(5.7 \times 10^4 \text{ m/s})^2}{\left(\frac{0.0068 \text{ m}}{2}\right)} = 9.56 \times 10^{11} \text{ m/s}^2$$

$$(c) a = \frac{F}{m} \Rightarrow \frac{v^2}{r} = \frac{qvB}{m} \Rightarrow B = \frac{ma}{qv} = \frac{(2.5 \times 10^{-26} \text{ kg})(9.56 \times 10^{11} \text{ m/s}^2)}{(1.6 \times 10^{-19} \text{ C})(5.7 \times 10^4 \text{ m/s})}$$

$$B = 2.62 \text{ T}$$

$$(d) r = \frac{mv}{qB} = \frac{(1.75 \times 10^{-26} \text{ kg})(5.7 \times 10^4 \text{ m/s})}{(1.6 \times 10^{-19} \text{ C})(2.62 \text{ T})} = 2.38 \times 10^{-3} \text{ m} = 2.38 \text{ mm}$$

IT INTERSECTS THE BARRIER AT $y = -2r = -4.76 \text{ mm}$.

Some Potentially Useful Formulae and Constants:

$$\vec{F}_{12} = k_e \frac{q_1 q_2}{r^2} \hat{r}_{12}$$

$$U_{12} = k_e \frac{q_1 q_2}{r_{12}}$$

$$\vec{E} = k_e \frac{q}{r^2} \hat{r}$$

$$\Delta U = -q \int_A^B \vec{E} \cdot d\vec{s}$$

$$\vec{E} = k_e \sum_i \frac{q_i}{r_i^2} \hat{r}_i$$

$$\Delta V = V_B - V_A = - \int_A^B \vec{E} \cdot d\vec{s}$$

$$\vec{E} = k_e \int \frac{dq}{r^2} \hat{r}$$

$$\Delta U = q\Delta V$$

$$\Phi_E = \int \vec{E} \cdot d\vec{A}$$

$$\vec{E} = - \left(\frac{dV}{dx} \hat{i} + \frac{dV}{dy} \hat{j} + \frac{dV}{dz} \hat{k} \right)$$

$$\Phi_E = \frac{q_{\text{inside}}}{\epsilon_0}$$

$$R = \frac{\Delta V}{I}$$

$$V = k_e \frac{q}{r}$$

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

$$V = k_e \sum_i \frac{q_i}{r_i}$$

$$a_r = \frac{v^2}{r}$$

$$V = k_e \int \frac{dq}{r}$$

$$V_{\text{sphere}} = \frac{4}{3} \pi r^3$$

$$A_{\text{sphere}} = 4\pi r^2$$

Physical constants:

$$k_e = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2$$

$$e = 1.602 \times 10^{-19} \text{ C}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 / \text{N} \cdot \text{m}^2$$

$$m_e = 9.11 \times 10^{-31} \text{ kg}$$

Mathematical formulae:

$$\int \frac{dr}{r^2} = -\frac{1}{r}$$

$$\int \frac{dx}{(x^2 + y^2)^{3/2}} = \frac{x}{y^2 \sqrt{x^2 + y^2}}$$

$$\int \frac{dx}{x} = \ln x$$

$$\int \frac{x dx}{(x^2 + y^2)^{3/2}} = -\frac{1}{\sqrt{x^2 + y^2}}$$

$$\int \frac{dx}{\sqrt{x^2 + y^2}} = \ln \left[x + \sqrt{x^2 + y^2} \right]$$

$$\vec{A} \cdot \vec{B} = AB \cos \theta = A_x B_x + A_y B_y + A_z B_z$$

$$\vec{A} \times \vec{B} = (A_y B_z - A_z B_y) \hat{i} + (A_z B_x - A_x B_z) \hat{j} + (A_x B_y - A_y B_x) \hat{k}$$