

SECTION 1: MULTIPLE CHOICE. (answer IN THE BOX provided for each question)

[2] 1. A simple pendulum consisting of a mass m hanging from a thin string of length l oscillates with an angular frequency ω . Which of the following changes would result in a higher frequency?

B

- A. Reducing the mass.
- B. Reducing the length of the string.
- C. Reducing the amplitude of the oscillation.
- D. Taking the pendulum to a planet with weaker gravity.

[2] 2. A positive charge is positioned at the origin. The force on this charge points in the positive x direction. Which of the following arrangements of additional charges (magnitude = q) could account for this?

C

- A. There is a negative charge ($-q$) at $x = -2.0$ cm on the x -axis.
- B. There is a negative charge ($-q$) at $x = -2.0$ cm on the x -axis and a positive charge ($+q$) at $x = -4.0$ cm on the x -axis.
- C. There is a negative charge ($-q$) at $x = +2.0$ cm on the x -axis and a positive charge ($+q$) at $x = +4.0$ cm on the x -axis.
- D. There is a positive charge ($+q$) at $x = +2.0$ cm on the x -axis.

[2] 3. A transverse wave traveling on a string is described by the wave function $y(x,t) = (0.03 \text{ m})\cos(2.0x - 6.0t)$ where x is in metres and t is in seconds. Which of the following statements **IS FALSE**?

B

- A. The maximum transverse speed of an element on the string is 0.18 m/s.
- B. The wavelength of the wave is 0.5 m.
- C. The wave is traveling in the positive x direction with a speed of 3.0 m/s.
- D. The period of the wave is $\frac{\pi}{3}$ s.

- [2] 4. Two guitar strings are the same length, held at both ends with the same tension, and made of the same material. Both have circular cross-sections. The fundamental frequency for string 1 is 512 Hz. The fundamental frequency for string 2 is 256 Hz. Which of the following statements **IS TRUE**?

A

- A. The diameter of string 2 is **two times** the diameter of string 1.
- B. The mass per unit length of string 2 is **half** the mass per unit length of string 1.
- C. The diameter of string 2 is **half** the diameter of string 1.
- D. None of the above statements can be true.

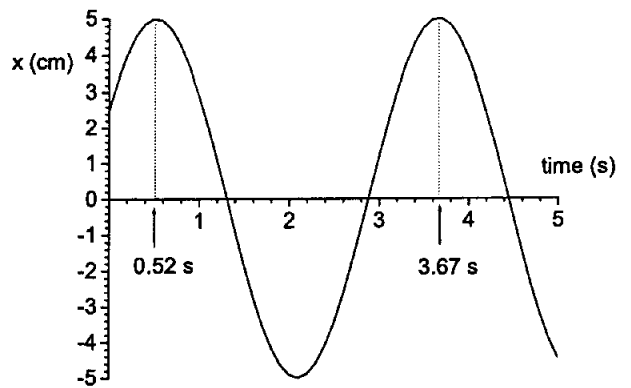
- [2] 5. A sound produced by exciting standing waves in a tube filled with air is found to contain all harmonics (i.e. $f_1, 2f_1, 3f_1, \dots$) of the fundamental. If end effects are negligible, which of the following statements **IS TRUE**?

B

- A. The wavelength of the fundamental mode was **four times** the length of the tube.
- B. The wavelength of the fundamental mode was **two times** the length of the tube.
- C. The wavelength of the fundamental mode was **half** the length of the tube.
- D. The wavelength of the fundamental mode was **equal** to the length of the tube.

SECTION 2: DO ALL FOUR (4) QUESTIONS

[10] 6. The graph represents the motion of a 1.7-kg mass oscillating along the x -axis while attached to a spring?



- (a) What is the period of this motion?
- (b) What is the angular frequency of this motion?
- (c) Write an equation for this motion in the form $x(t) = A\cos(\omega t + \phi)$. (Hint: what is the value of $(\omega t + \phi)$ at $t = 0.52$ s?)
- (d) What is the maximum speed of this mass?
- (e) What is the total mechanical energy of this system?

(a) $T = 3.15$ s

(b) $\omega = \frac{2\pi}{T} = 1.99 \text{ rad/s}$

(c) $\omega t + \phi = 0$ at $t = 0.525$

$\therefore \phi = -1.99 \times 0.52 \text{ rad} = -1.04 \text{ rad}$

$\therefore x(t) = 5 \text{ cm} \cos(1.99 \frac{\text{rad}}{\text{s}} \times t - 1.04 \text{ rad})$

(d) $|v_{\text{max}}| = A\omega$

$= 5 \text{ cm} \times 1.99 \text{ rad/s}$

$= 9.95 \text{ cm/s}$

$= .0995 \text{ m/s}$

(e) $E_{\text{TOT}} = K_{\text{max}}$

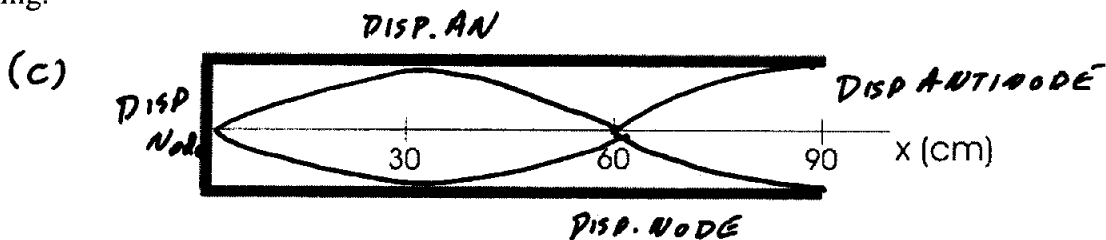
$= \frac{1}{2} m v_{\text{max}}^2$

$= \frac{1}{2} (1.7 \text{ kg}) (0.0995 \text{ m/s})^2$

$= 0.0084 \text{ J}$

[10] 7. A tube of length 90.0 cm is open at one end and closed at the other end. It is observed to resonate at 95.0 Hz and at 285 Hz but not at any frequencies between these two values.

- (a) What is the fundamental frequency for this tube? **Briefly** explain your reasoning.
- (b) What is the speed of sound in air under the conditions of this experiment?
- (c) Use the outline of the tube below to draw a graphical representation of displacement amplitude versus position along the tube for the 285 Hz mode in this tube. Clearly indicate displacement nodes and displacement antinodes on your drawing.



(a) TUBE CLOSED ONE END, OPEN OTHER
 - Adjacent resonances separated by $2 \times f_{\text{fundamental}}$

i.e.

fund. $\Rightarrow f_1$ ($\lambda = 4L$)
 3^{rd} harmonic $\Rightarrow 3f_1$ ($\lambda = \frac{4}{3}L$)

So: $2 \times f_{\text{fundamental}} = 285 \text{ Hz} - 95 \text{ Hz} = 190 \text{ Hz}$
 $\therefore f_{\text{fundamental}} = f_1 = 95 \text{ Hz}$

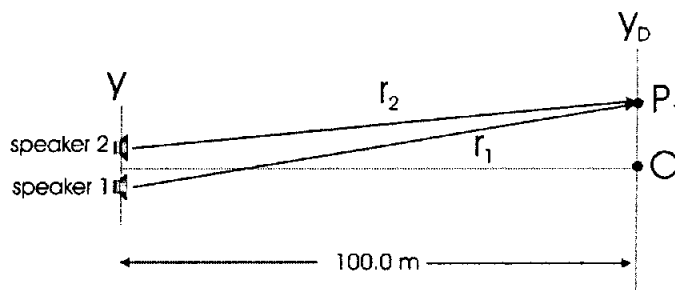
(b) $v = f \lambda$ for $f_{\text{fund}} = 95 \text{ Hz}$, $\lambda_{\text{fund}} = 4 \times L$

$\therefore \lambda_{\text{fundamental}} = 4 \times 0.9 \text{ m} = 3.6 \text{ m}$

$\therefore v_s = 95 \text{ Hz} \times 3.6 \text{ m}$
 $= 342 \text{ m/s}$

[10] 8. (a) Two speakers that emit identical sounds, in phase, are separated by a small distance along the y-axis. Briefly describe why sound intensity at some locations in front of the speakers is a maximum and, at others, is a minimum.

(b) A sound detector is set up so that it can move along line y_D parallel to the y-axis. (Assume that the speed of sound is 330 m/s.)



(i) With the speakers emitting at 256 Hz, the detector starts at point O and moves along line y_D . The detected sound starts at maximum intensity at O, then decreases to a minimum before increasing back to a maximum at P_1 . What is the difference, $r_1 - r_2$, between the distances from P_1 to each of the speakers?

(ii) The frequency is then changed to f_2 , and the experiment repeated. This time, as the detector is moved from O to P_1 , the detected sound intensity goes from a maximum at O to a minimum, back to a maximum, and finally back to a second minimum when the detector reaches P_1 . What is the frequency f_2 ?

(a) Waves leave speakers in phase:

- They interfere constructively (add to bigger amplitude) when phases at P_1 differ by $0, 2\pi, 4\pi, \dots$

This is when $\Delta r = |r_1 - r_2| = 0, \lambda, 2\lambda, \dots$

- They interfere destructively (cancel) when phases at P_1 differ by $\pi, 3\pi, 5\pi, \dots$

This is when $\Delta r = |r_1 - r_2| = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \dots$

(b)(i) At P_1 (first maximum), $r_1 - r_2 = \lambda$

$$\text{But } \lambda = \frac{v}{f} = \frac{330 \text{ m/s}}{256 \text{ Hz}} = 1.29 \text{ m.}$$

$$\therefore \text{At } P_1, r_1 - r_2 = 1.29 \text{ m.}$$

(ii) For f_2 , P_1 is at SECOND minimum.

$$\text{So } r_1 - r_2 = \frac{3\lambda}{2} \quad (\text{for } P_2)$$

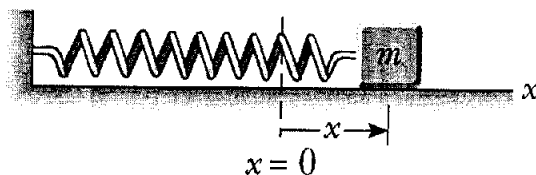
We know $r_1 - r_2 = 1.29 \text{ m.}$

$$\text{So } \frac{3\lambda}{2} = 1.29 \text{ m}$$

$$\therefore \lambda = \frac{2 \times 1.29 \text{ m}}{3} = 0.859 \text{ m.}$$

$$\therefore f = \frac{v}{\lambda} = \frac{330 \text{ m/s}}{0.859 \text{ m}} = 384 \text{ Hz}$$

- [10] 9. A block is attached to a spring and is oscillating on a frictionless horizontal surface with an amplitude of 0.05 m. The maximum speed of the block as it oscillates is 0.45 m/s. The force constant of the spring is 12.0 N/m.



- (a) What is the period of the oscillation?
 (b) What is the mass of the block?
 (c) What is the speed of the block when it is 3.53 cm from its equilibrium position?

(a) Know A and v_{max} so we can get ω from

$$v_{max} = A\omega$$

And then get period from $\omega = \frac{2\pi}{T}$

$$\Rightarrow v_{max} = A\omega$$

$$\therefore \omega = \frac{v_{max}}{A} = \frac{0.45 \text{ m/s}}{0.05 \text{ m}} = 9 \text{ rad/s}$$

$$\therefore T = \frac{2\pi}{\omega} = \frac{2\pi}{9} \text{ s} = 0.698 \text{ s}$$

(b) $\omega = \sqrt{\frac{k}{m}}$

$$\therefore m = \frac{k}{\omega^2} = \frac{12.0 \text{ N/m}}{(9 \text{ rad/s})^2} = 0.148 \text{ kg}$$

(c) know k and A so we can get $E_{TOT} = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$

$$\begin{aligned} \therefore E_{TOT} &= \frac{1}{2}kA^2 \\ &= \frac{1}{2}12 \text{ N/m} \times (0.05 \text{ m})^2 \\ &= 0.015 \text{ J} \end{aligned}$$

$$\text{But } E_{TOT} = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

$$\begin{aligned} \therefore 0.015 \text{ J} &= \frac{1}{2}(12 \text{ N/m})(0.0353 \text{ m})^2 + \frac{1}{2}(0.148 \text{ kg})v^2 \\ &= 0.00748 \text{ J} + 0.074 \text{ kg} \times v^2 \end{aligned}$$

$$\therefore v^2 = \frac{0.015 - 0.00748 \text{ J}}{0.074 \text{ kg}} = 0.102 \text{ m}^2/\text{s}^2$$

$$\therefore v = 0.319 \text{ m/s}$$