

- [2] 4. Two guitar strings are the same length and are held at both ends with the same tension. The fundamental frequency of one string is 512 Hz and when they are played together, a beat frequency of 13 Hz is heard. Which of the following statements **IS TRUE**?

C

- A. Transverse waves must travel along both strings with the same speed.
 $f = v/\lambda$. λ is same. T is same so v must be different.
- B. The wavelengths of the fundamental modes on the two strings must be different.
No. Same length strings. $\lambda_f = 2L$.
- C. The two strings must have different masses.
Yes. wave speed is different so must have different m .
- D. The fundamental frequency of the second string must be 525 Hz.
No. Second string could be higher or lower

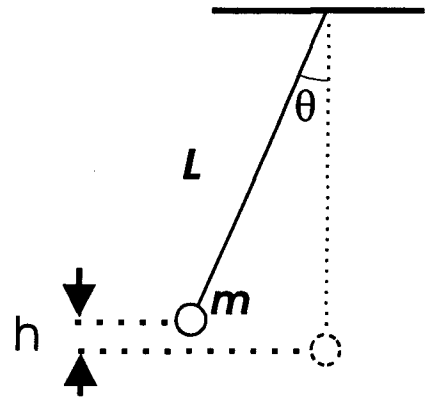
- [2] 5. An electrical oscillator connected to a speaker can be used to produce a sinusoidal sound wave at a frequency of 440 Hz. A trumpet can be used to produce a non-sinusoidal sound wave with a fundamental frequency of 440 Hz. Both of these sound waves are analyzed. Which of the following statements **IS TRUE**?

C

- A. The highest frequency contained in the non-sinusoidal wave produced by the trumpet is the fundamental.
- B. The sinusoidal wave contains the fundamental and a series of harmonics.
- C. The non-sinusoidal wave produced by the trumpet contains the fundamental and a series of harmonics.
- D. The sinusoidal wave contains all frequencies less than the fundamental.

SECTION 2: DO ALL FOUR (4) QUESTIONS

[10] 6. A simple pendulum is constructed by suspending a mass $m = 2.3 \text{ kg}$ from a massless string of length l . After the mass is disturbed from equilibrium, its angular position is found to be given by $\theta(t) = (0.15 \text{ rad})\cos(3.6t + \pi)$ where the angle is measured from the vertical and counterclockwise is taken to be positive.



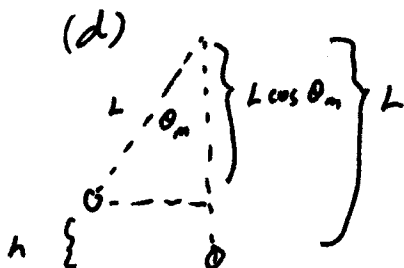
- (a) What is the angular frequency of this pendulum?
- (b) What is the period of the pendulum's motion?
- (c) What is the length of the pendulum?
- (d) What is the maximum height h , above equilibrium, reached by this mass?
- (e) What is the total mechanical energy of this system?
- (f) What is the maximum speed of the mass?

$$\theta(t) = \underset{\theta_{\max}}{0.15 \text{ rad}} \cos(\underset{\omega}{3.6t} + \pi)$$

(a) $\omega = 3.6 \text{ rad/s}$

(b) $T = 2\pi/\omega = 1.75 \text{ s}$

(c) $\omega = \sqrt{g/L} \quad \therefore L = g/\omega^2 = \frac{9.8 \text{ m/s}^2}{(3.6 \text{ s}^{-1})^2} = 0.756 \text{ m}$

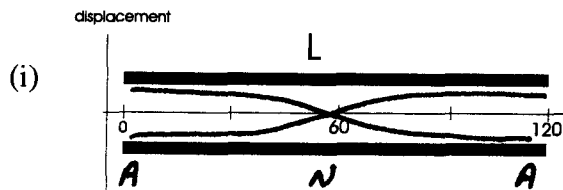
(d)  $\Rightarrow h = L - L \cos \theta_{\max}$
 $= L(1 - \cos \theta_{\max})$
 $= 0.756 \text{ m} \times (1 - \cos(0.15 \text{ rad}))$
 $= 8.49 \times 10^{-3} \text{ m}$

(e) $E_{\text{mech}} = U_{\max} = mgh = 2.3 \text{ kg} \times 9.8 \text{ m/s}^2 \times 8.49 \times 10^{-3} \text{ m}$
 $= 0.191 \text{ J}$

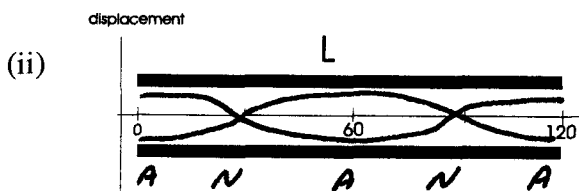
(f) $E_{\text{mech}} = \frac{1}{2} m v_{\max}^2$
 $\therefore v_{\max} = \sqrt{\frac{2 E_{\text{mech}}}{m}} = \sqrt{\frac{2 \times 0.1915}{2.3 \text{ kg}}} = 0.408 \text{ m/s}$

[10] 7. (a) Consider two tubes of length L . One is open at both ends and the other is open at one end and closed at the other. If λ_f is the wavelength of the fundamental mode in a given tube, for which tube is $\lambda_f = 2L$ and for which tube is $\lambda_f = 4L$?

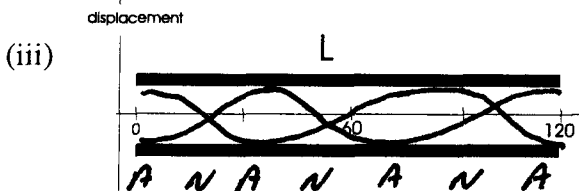
(b) A tube of length 120.0 cm is open at both ends. Using the tube outlines below, carefully draw and label graphical representations of displacement amplitude versus position along the tube for the fundamental (i) and the next two **allowed** harmonics in this tube (ii), (iii). Clearly indicate displacement nodes (N) and displacement antinodes (A) on your drawing.



$x(\text{cm})$ Fundamental. $\lambda = 2L$



$x(\text{cm})$ 2nd Harmonic. $\lambda = L$



$x(\text{cm})$ 3rd Harmonic: $\lambda = \frac{2}{3}L$

(c) Two of the frequencies (but not necessarily the lowest two) at which this tube resonates are 286 Hz and 429 Hz. There is no resonance between 286 Hz and 429 Hz. What is the frequency of the fundamental mode?

(d) What is the speed of sound in air under the conditions of this experiment?

(e) Which of the modes drawn in part (b) corresponds to the 286 Hz mode?

(a) for tube open both ends, $\lambda_f = 2L$
 for tube open one end, closed one end, $\lambda_f = 4L$

(c) for tube open both ends

$$f_{n+1} - f_n = f_1$$

$$\therefore f_1 = 429 \text{ Hz} - 286 \text{ Hz} = 143 \text{ Hz}$$

(d) $v = f \lambda \quad \therefore v = f_1 \times \lambda_f = 143 \text{ Hz} \times (2 \times 1.2 \text{ m}) = 343 \text{ m/s}$

(e) 286 Hz is second harmonic

\therefore (ii) is mode corresponding to 286 Hz.

[10] 8. Waves traveling in the x -direction along a **very** long string are seen to propagate with a speed of 55.0 m/s. The linear density (mass per unit length) of the string is 0.007 kg/m.

- (a) What is the tension in the string?
- (b) A ribbon wrapped around part of the string is seen to undergo simple harmonic motion with a period of 0.064 s. What is the frequency of the wave?
- (c) What is the wavelength of the wave?
- (d) If the amplitude of the wave is 0.039 m, what is the maximum transverse speed of the ribbon?
- (e) Write an expression for the wavefunction in the form $y(x,t) = A \sin(kx - \omega t + \phi)$. Assume that $y(0,0) = 0$ and $v_y(0,0) < 0$.

$$(a) \quad v = \sqrt{\frac{T}{\mu}} \quad \therefore T = v^2 \mu = (55 \frac{m}{s})^2 \times 0.007 \frac{kg}{m} = 21.2 N$$

$$(b) \quad f = \frac{1}{\text{period}} = \frac{1}{(0.064s)} = 15.6 \text{ Hz}$$

$$(c) \quad v = f \lambda \quad \therefore \lambda = \frac{v}{f} = \frac{55 \frac{m}{s}}{15.6 \text{ Hz}} = 3.53 \text{ m}$$

$$(d) \quad y = A \sin(kx - \omega t + \phi)$$
$$v_y = -A\omega \cos(kx - \omega t + \phi)$$
$$\therefore |v_{y \text{ max}}| = A\omega$$
$$= 0.039 \text{ m} \times 2\pi \times 15.6 \text{ Hz}$$
$$= 3.82 \frac{m}{s}$$

$$(e) \quad y(0,0) = A \sin \phi = 0$$
$$\therefore \sin \phi = 0 \Rightarrow \phi = 0 \text{ or } \phi = \pi$$
$$v_y(0,0) = -A\omega \cos \phi < 0$$
$$\therefore \cos \phi > 0$$
$$\therefore \text{must be } \phi = 0$$

$$k = \frac{2\pi}{\lambda} = 1.78 \text{ m}^{-1} \quad \omega = 2\pi f = 98.0 \text{ s}^{-1}$$

$$\therefore y(x,t) = (0.039 \text{ m}) \sin(1.78 \text{ m}^{-1} x - 98.5 \text{ s}^{-1} t)$$

[10] 9. One charge, $q_A = +2 \mu\text{C}$, is located at the origin. A second charge, $q_B = -3 \mu\text{C}$, is located at $x = 1.0 \text{ m}$, $y = 2.0 \text{ m}$. Point **P** is located on the y -axis at $y = 2.0 \text{ m}$.

- Calculate the contribution \vec{E}_A to the electric field at **P** due to q_A and the contribution \vec{E}_B to the electric field at **P** due to q_B .
- On the diagram, draw vectors representing \vec{E}_A and \vec{E}_B .
- Calculate the total electric field at **P**. Express your answer in unit vector notation.
- What would be the magnitude of the force on a charge $q_C = -0.15 \mu\text{C}$ placed at point **P**?

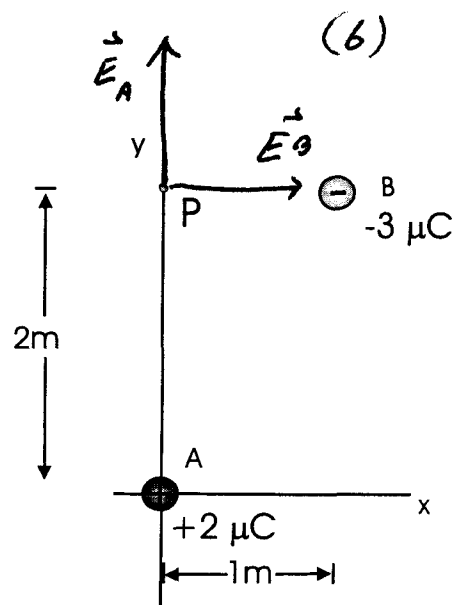
$$(a) \vec{E}_A = k_e \frac{2 \times 10^{-6} \text{ C}}{(2 \text{ m})^2} \hat{j} = 4.5 \times 10^3 \text{ N/C } \hat{j}$$

$$\vec{E}_B = k_e \frac{3 \times 10^{-6} \text{ C}}{(1 \text{ m})^2} \hat{i} = 2.7 \times 10^4 \text{ N/C } \hat{i}$$

$$(c) \vec{E} = \vec{E}_A + \vec{E}_B \\ = 2.7 \times 10^4 \text{ N/C } \hat{i} + 4.5 \times 10^3 \text{ N/C } \hat{j}$$

$$(d) \vec{F} = q \vec{E} \\ = (-0.15 \times 10^{-6} \text{ C}) \times (2.7 \times 10^4 \text{ N/C } \hat{i} + 4.5 \times 10^3 \text{ N/C } \hat{j}) \\ = -4.05 \times 10^{-3} \text{ N } \hat{i} - 6.75 \times 10^{-4} \text{ N } \hat{j}$$

$$|\vec{F}| = \sqrt{F_x^2 + F_y^2} \\ = \sqrt{(4.05 \times 10^{-3} \text{ N})^2 + (6.75 \times 10^{-4} \text{ N})^2} \\ = 4.11 \times 10^{-3} \text{ N}$$



Some Potentially Useful Formulae and Constants:

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

$$F_x = -k_{\text{spring}} x \quad (\text{Hooke's Law})$$

$$\omega = \frac{2\pi}{T} \quad (\text{angular frequency})$$

$$\omega^2 = \frac{k_{\text{spring}}}{m}$$

$$\omega^2 = \frac{g}{L}$$

$$\omega^2 = \frac{mgd}{I}$$

$$k = \frac{2\pi}{\lambda} \quad (\text{angular wave number})$$

$$v = f\lambda \quad (\text{wave speed})$$

$$v = \sqrt{\frac{T}{\mu}}$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}$$

$$\vec{F}_{12} = k_e \frac{q_1 q_2}{r^2} \hat{r}_{12}$$

$$\vec{E} = k_e \frac{q}{r^2} \hat{r}$$

$$\vec{E} = k_e \sum_i \frac{q_i}{r_i^2} \hat{r}_i$$

$$v_{\text{sound}} = 331 \text{ m/s} + 0.6 \frac{\text{m}}{\text{s} \cdot ^\circ\text{C}} \times T_{\circ\text{C}}$$

Physical constants:

$$k_e = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2$$

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 / \text{N} \cdot \text{m}^2$$

$$e = 1.60 \times 10^{-19} \text{ C}$$

$$g = 9.81 \text{ m/s}^2$$