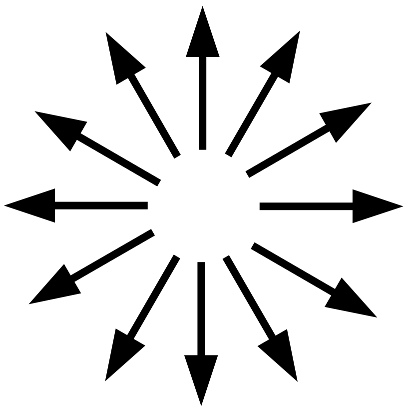
Electric Field Lines

- Electric forces

- Electric fields:
  - Electric field lines emanate from positive charges
  - Electric field lines disappear at negative charges

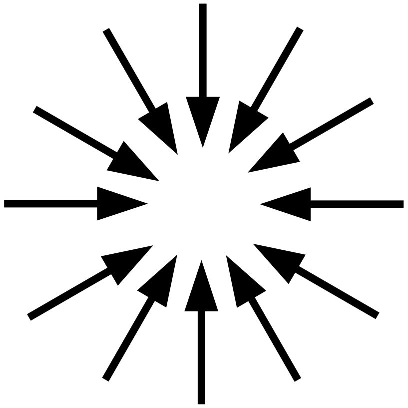


If you see a bunch of field lines emanating from a closed surface, there must be a positive charge inside!

Electric Field Lines

- Electric forces

- Electric fields:
  - Electric field lines emanate from positive charges
  - Electric field lines disappear at negative charges



If you see a bunch of field lines ending on a closed surface, there must be a negative charge inside!

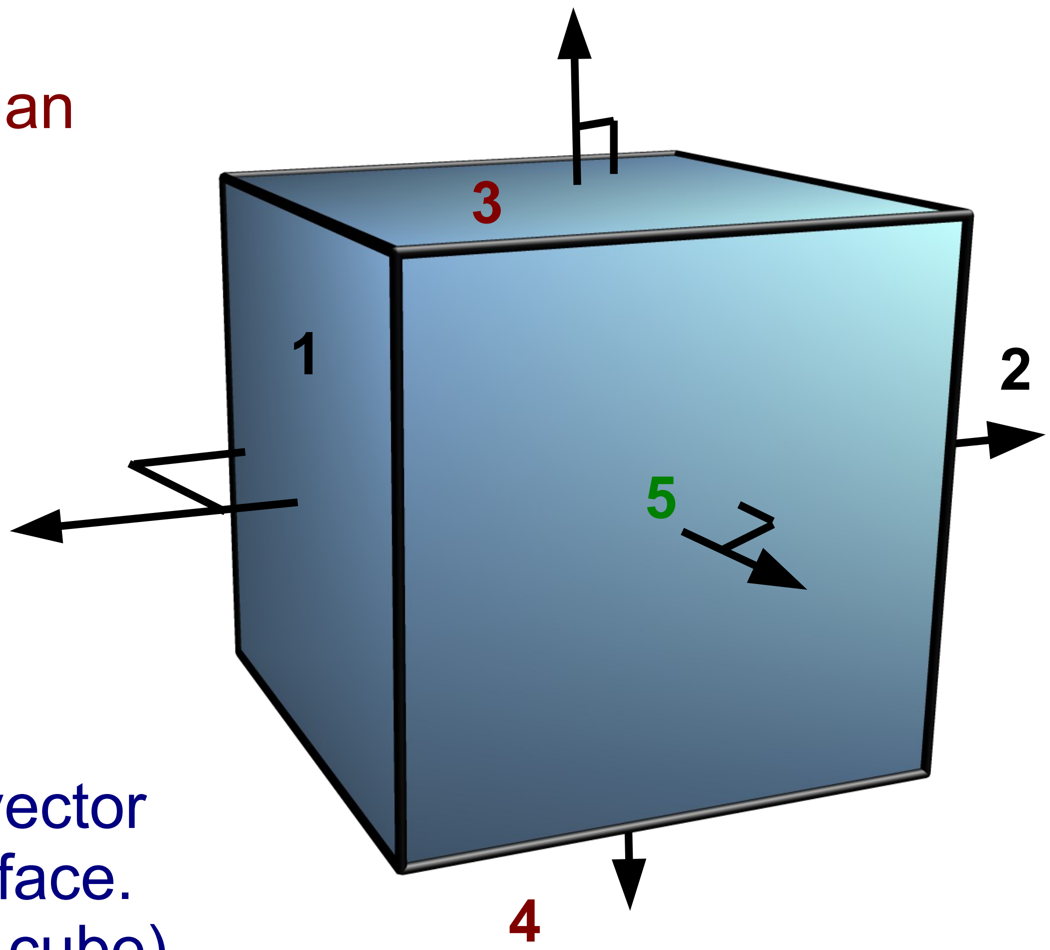
Electric Flux

Electric flux is a quantity that is proportional to the number of field lines passing though a given area.

This is a loose definition. We will make this definition more concrete. The concept of electric flux is very important. It will lead us to the most important result in electrostatics:
by looking at the electric flux through a closed surface, you can tell how much charge is enclosed within it

Electric Flux

Step 1
Define a “surface normal” to an element of area.



Consider this cube.
It has 6 surfaces:
1 and **2** (left and right)
3 and **4** (top and bottom)
5 and **6** (front and back)

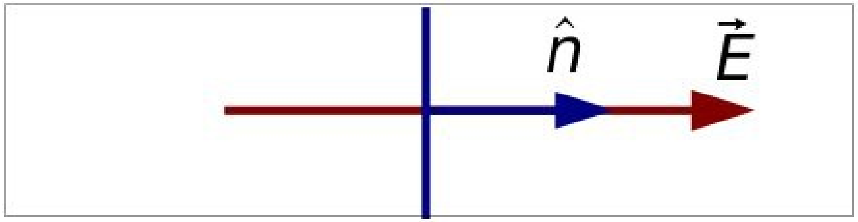
The “surface normal” is a unit vector that is perpendicular to the surface.
If the surface is closed (like this cube) the unit vector always points outward.

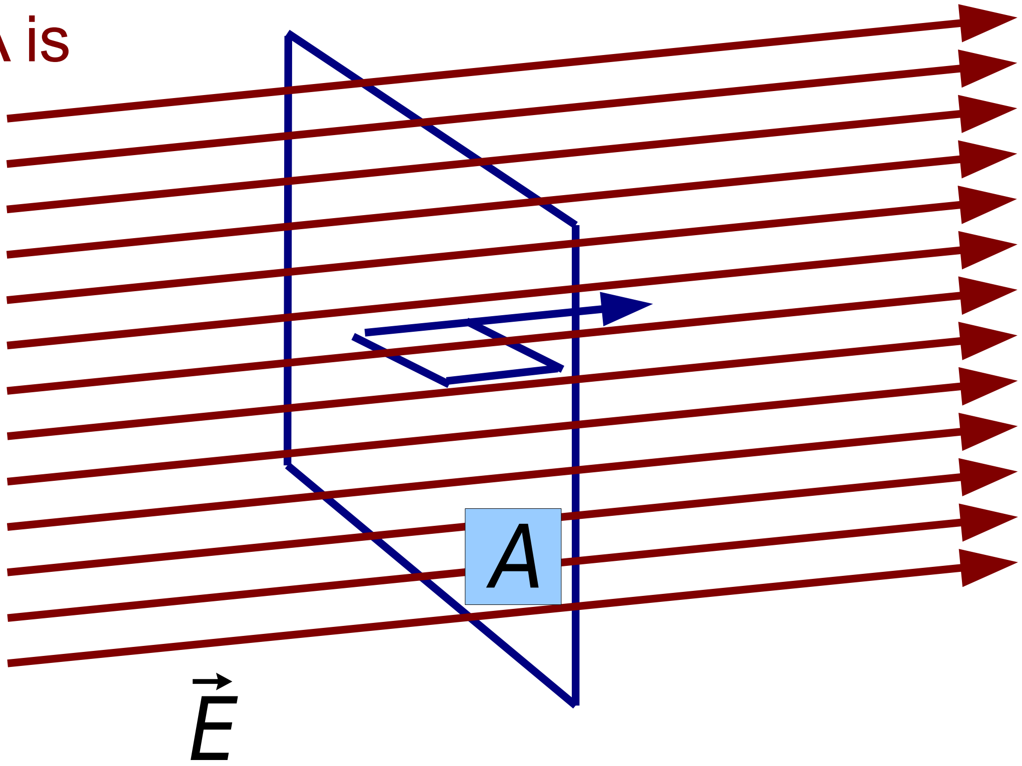
Electric Flux

Step 2

If the "surface normal" is parallel to the direction of the electric field, then the flux through an area A is

$$\Phi_E = EA$$

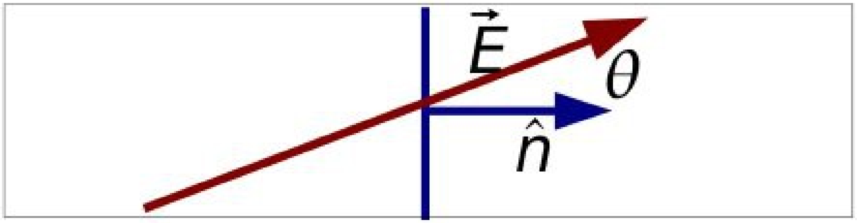


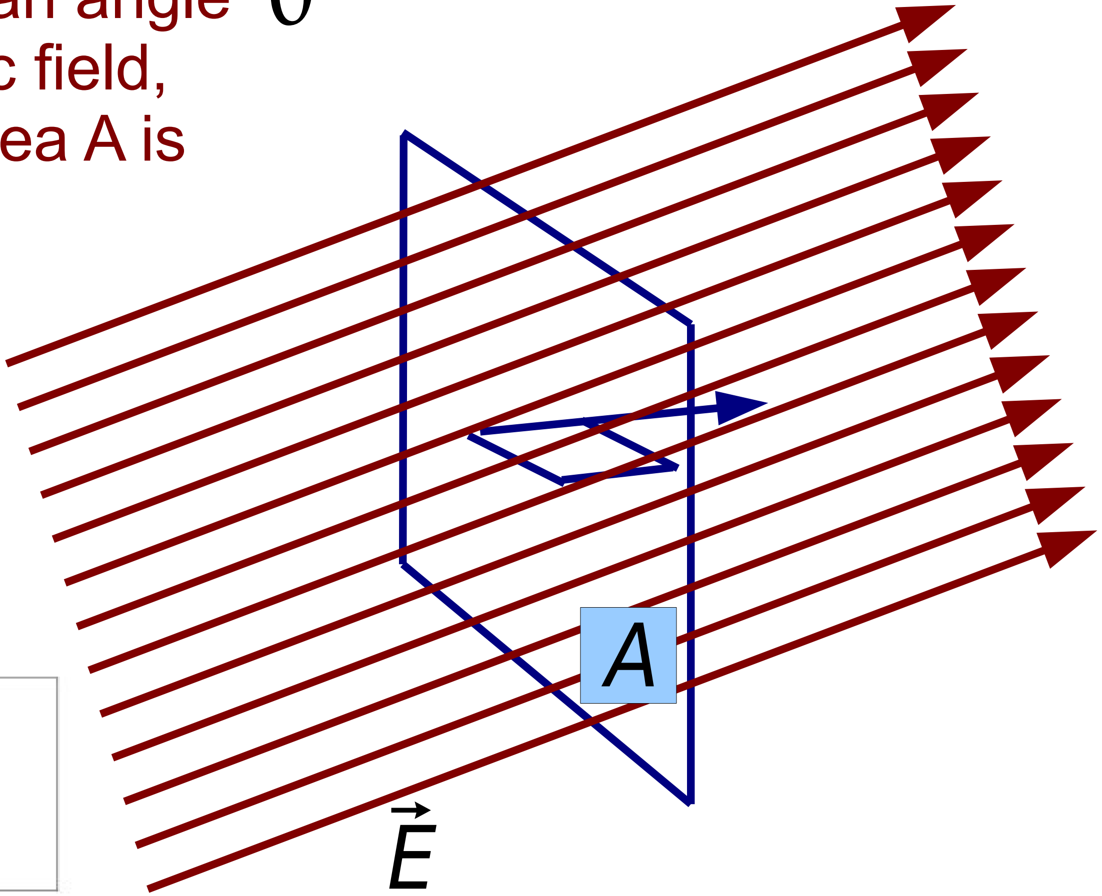


Electric Flux

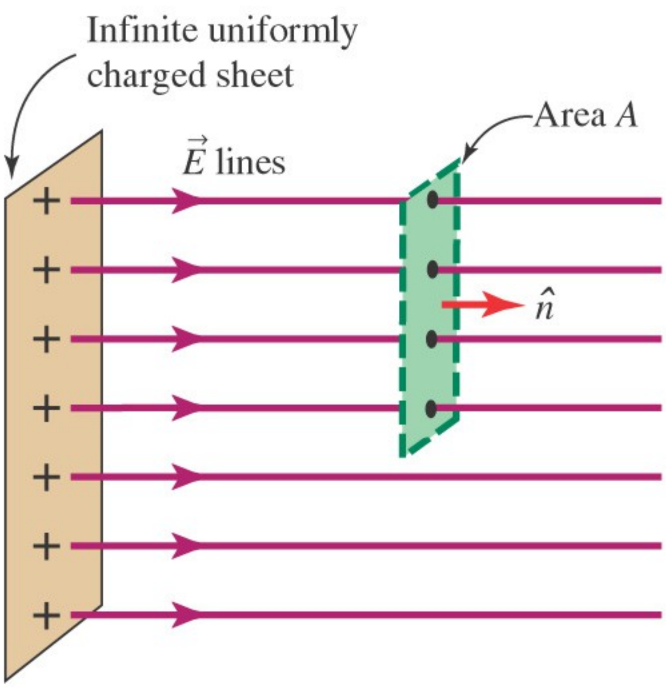
Step 2
If the “surface normal” is at an angle θ
the direction of the electric field,
then the flux through an area A is

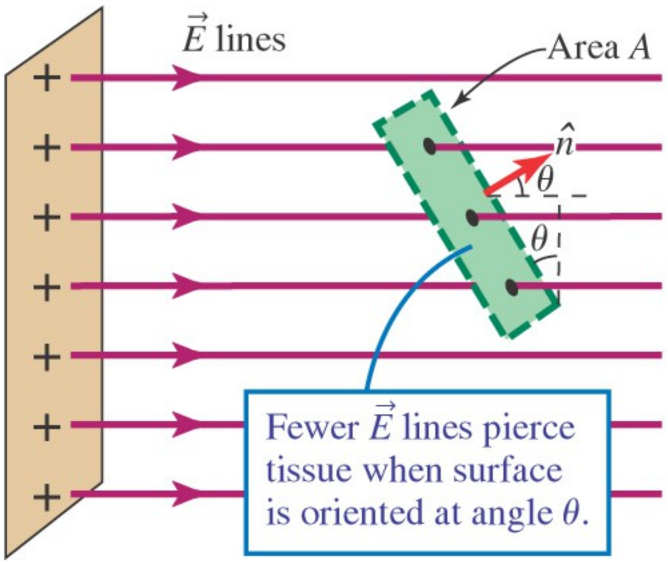
$$\Phi_E = E\,A\cos\theta$$





Electric Flux





$$d\Phi_E = \vec{E} . \vec{dA} = \vec{E} . \hat{n}\, dA$$

Electric Flux

Step 3

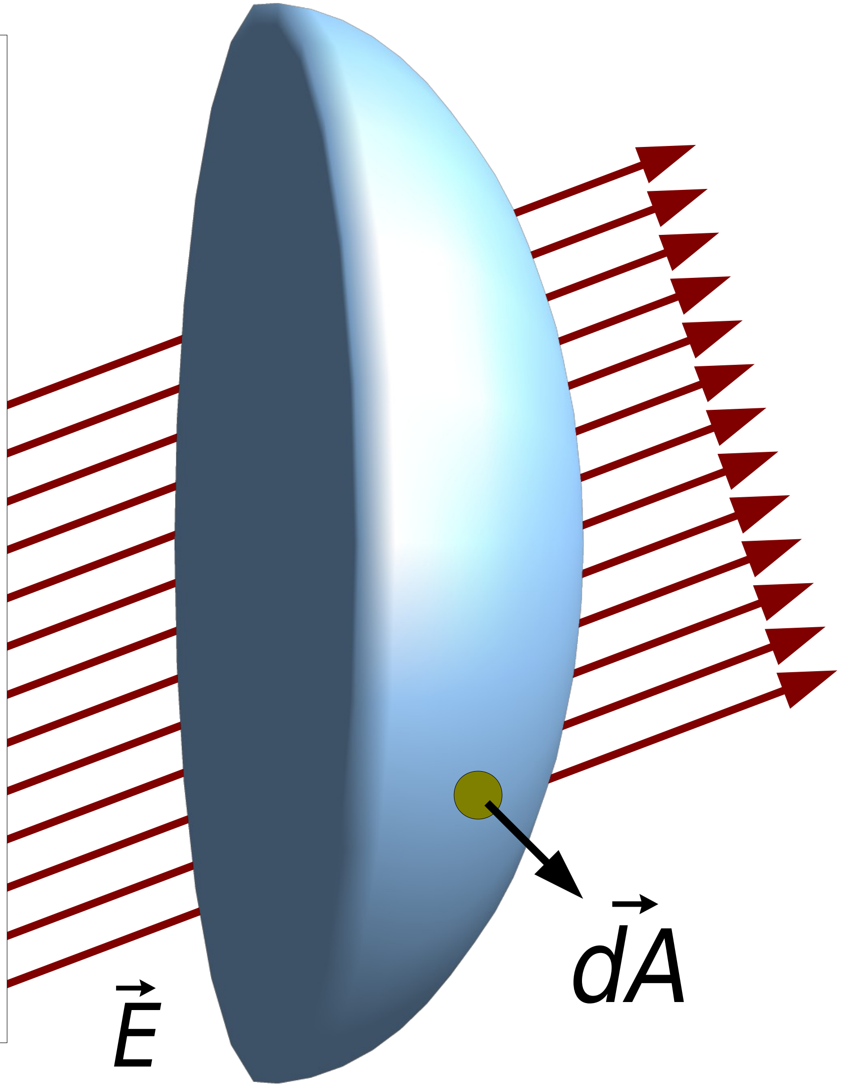
How to deal with curved surfaces ?

Define an infinitesimal area $d\vec{A}$

$$d\vec{A} = \hat{n}\, dA$$

The flux through this area is

$$d\Phi_E = \vec{E} . d\vec{A} = \vec{E} . \hat{n}\, dA$$



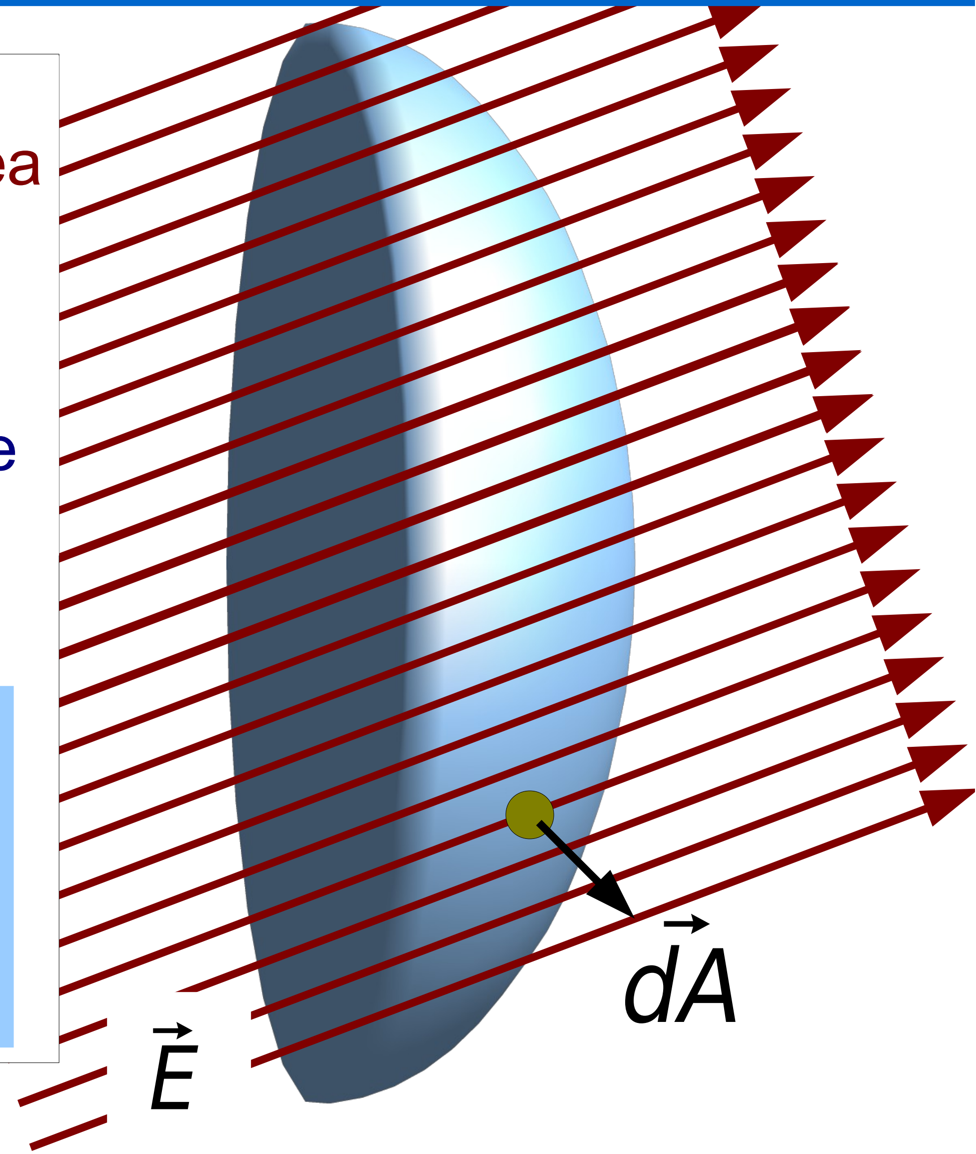
Electric Flux

Step 4

To get the flux through the entire area we must integrate over the entire surface

The total electric flux over the entire curved surface is:

$$\Phi_E = \int_{surface} \vec{E} \, . \, \vec{dA}$$
$$= \int_{surface} \vec{E} \, . \, \hat{n} \, dA$$



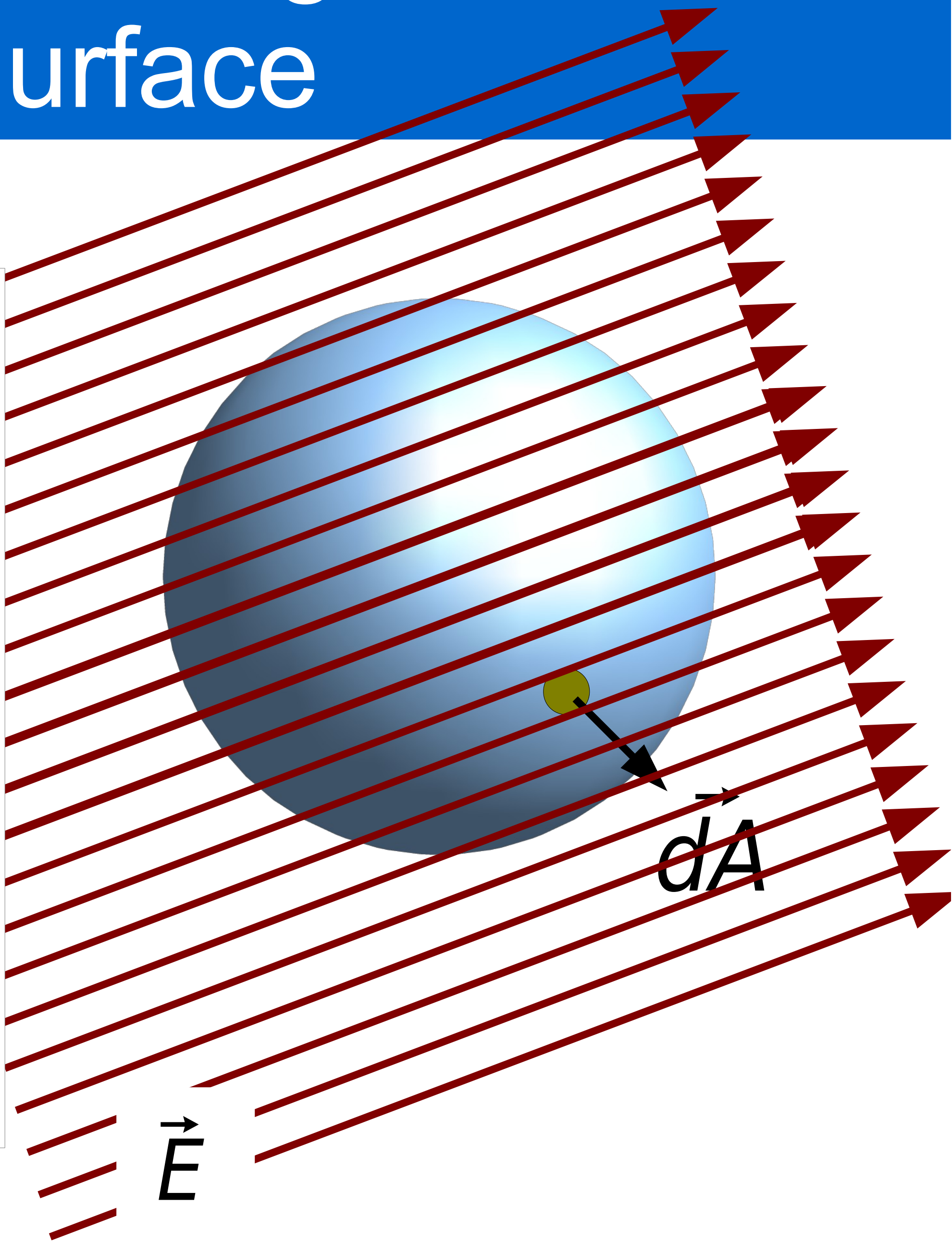
Electric Flux Through A Closed Surface

Lets break this down into three cases:

1. surface whose normals are at angles between $-\pi/2$ and $\pi/2$ with respect to the electric field direction.

2. surface whose normals are at an angles of $\pi/2$ with respect to the electric field direction.

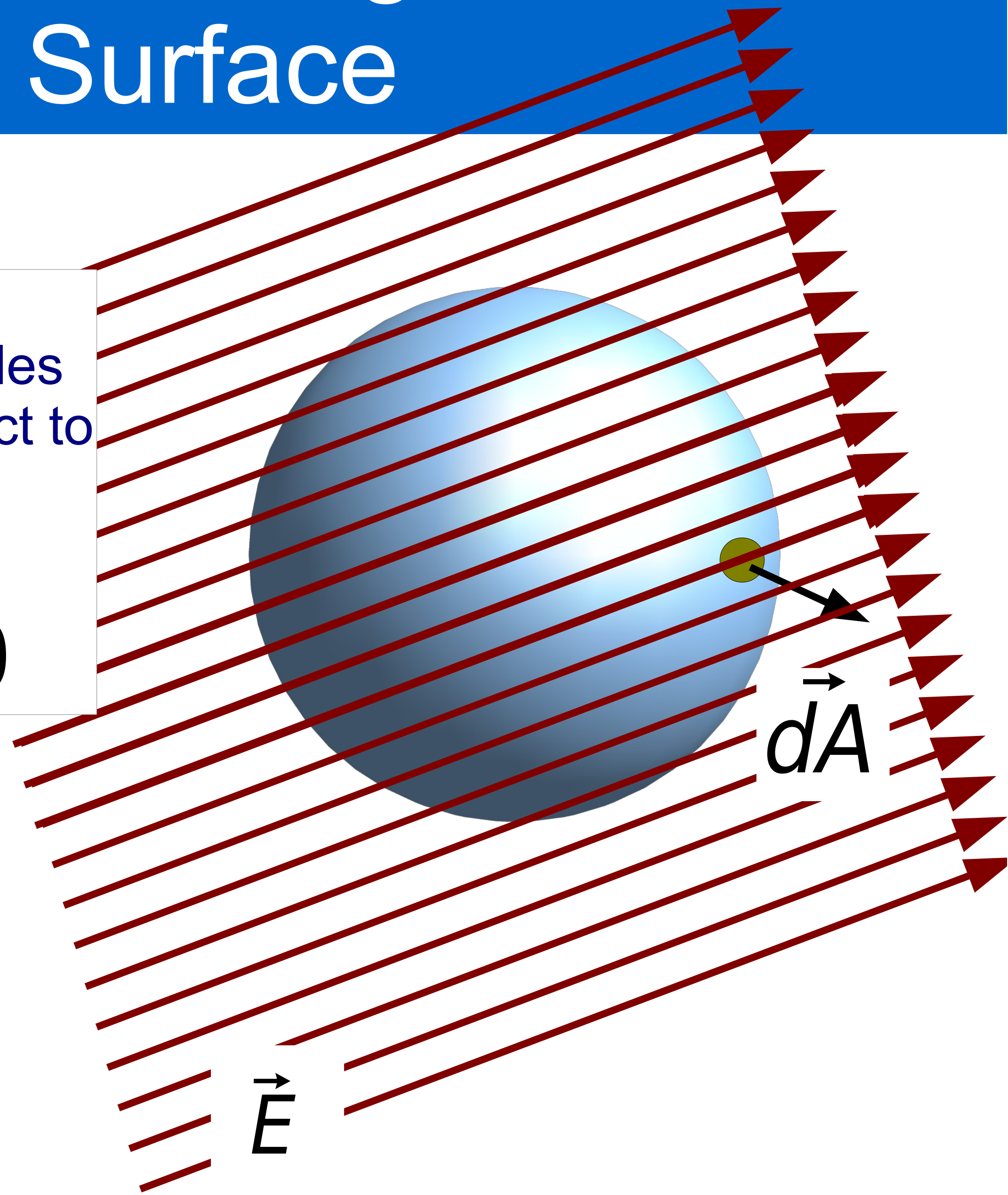
3. surface whose normals are at angles between $\pi/2$ and $3\pi/2$ with respect to the electric field direction.



Electric Flux Through A Closed Surface

1. surface whose normals are at angles between $-\pi/2$ and $\pi/2$ with respect to the electric field direction.

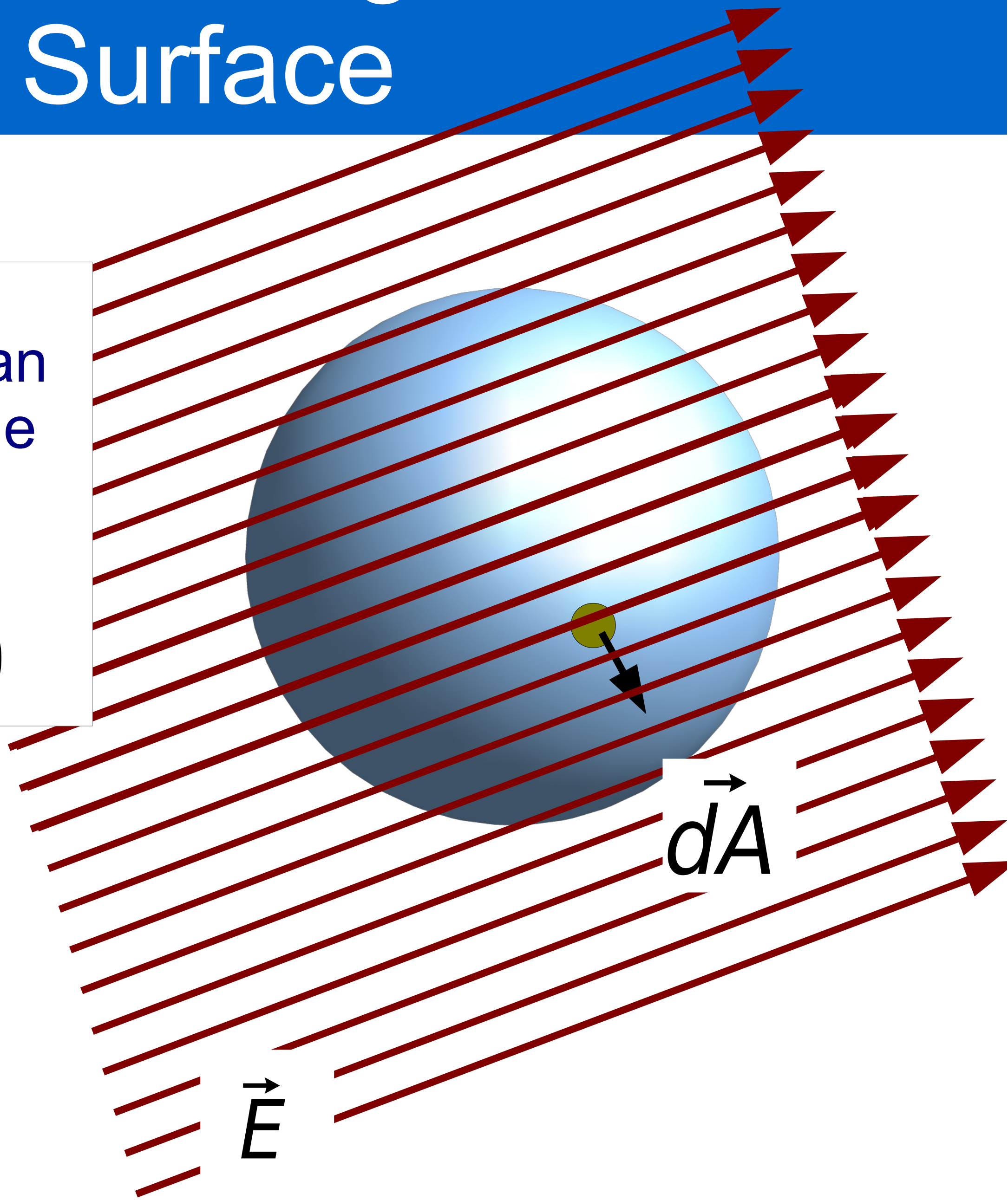
$$\vec{E}\,.\,\vec{dA} = E\,dA\cos\theta > 0$$



Electric Flux Through A Closed Surface

2. surface whose normals are at an angles of $\pi/2$ with respect to the electric field direction.

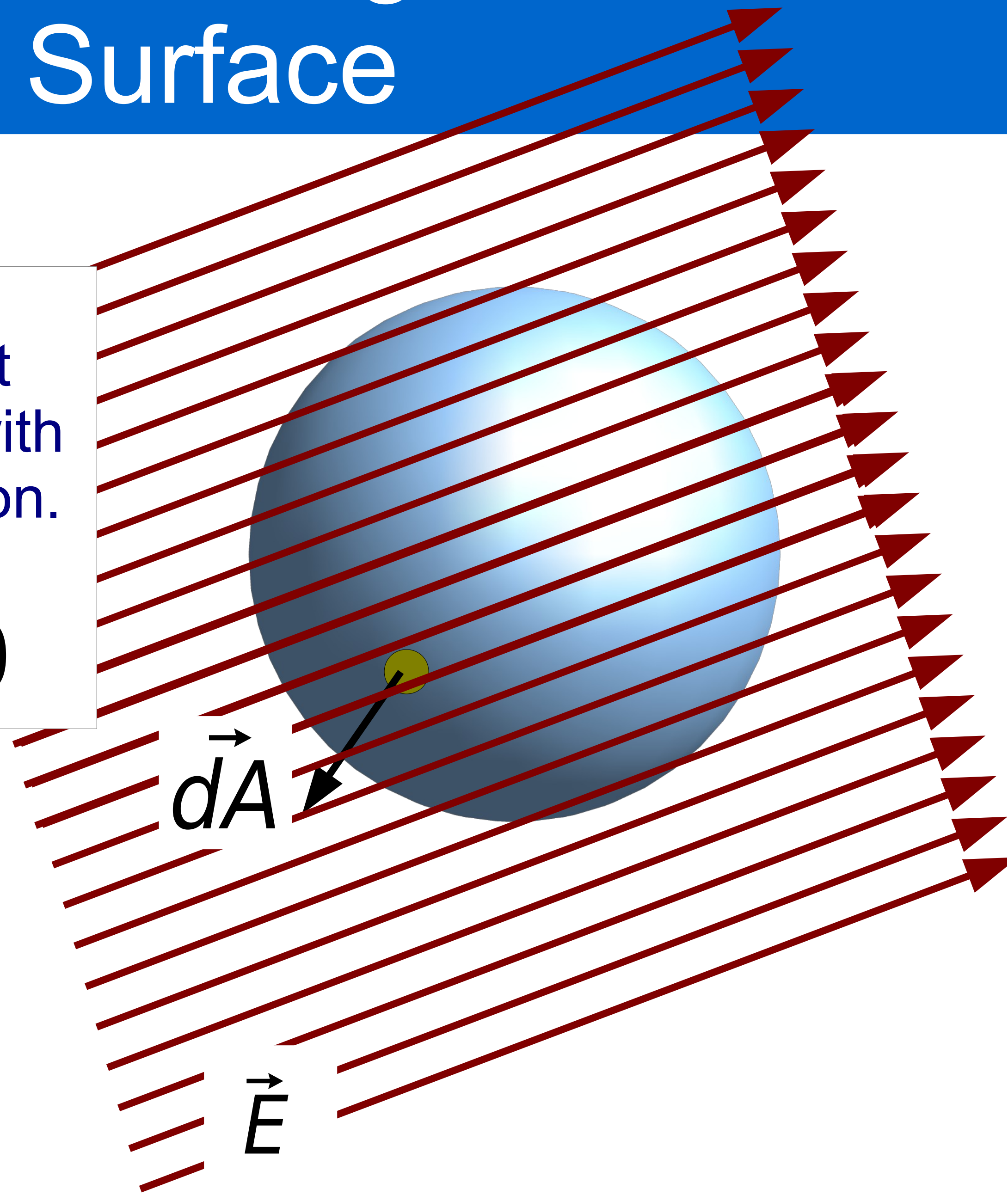
$$\vec{E}\,.\,\vec{dA} = E\,dA\cos\theta = 0$$



Electric Flux Through A Closed Surface

3. surface whose normals are at angles between $\pi/2$ and $3\pi/2$ with respect to the electric field direction.

$$\vec{E}\,.\,\vec{dA} = E\,dA\cos\theta < 0$$



Electric Flux Through A Closed Surface

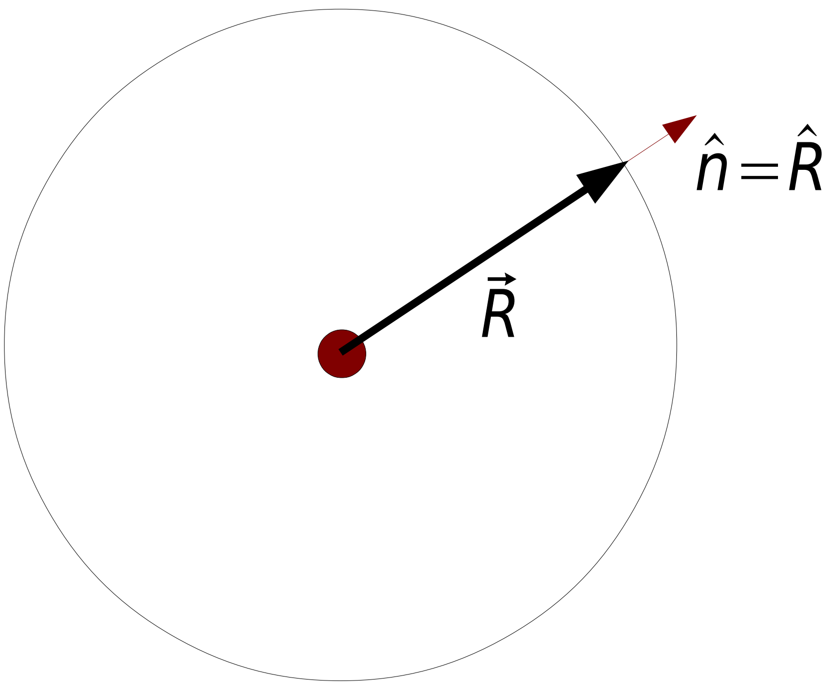
The total electric flux through a closed surface will therefore be a sum over positive, negative and zero contributions.

To get the total electric flux, we must do the entire integral.

$$\Phi_E = \int_{closed\ surface} \vec{E} \,.\, d\vec{A}$$
$$= \oint \vec{E} \,.\, \hat{n}\, dA$$

Electric Flux Through A Closed Surface

Lets consider the flux through the surface of a sphere (radius R) that contains a charge q at the centre.



$$\Phi_E = \oint \vec{E} \,.\, \hat{n}\, dA$$

$$= \oint \frac{k_e q}{R^2} \hat{R} \,.\, \hat{n}\, dA$$

$$= \frac{k_e q}{R^2} \oint dA$$

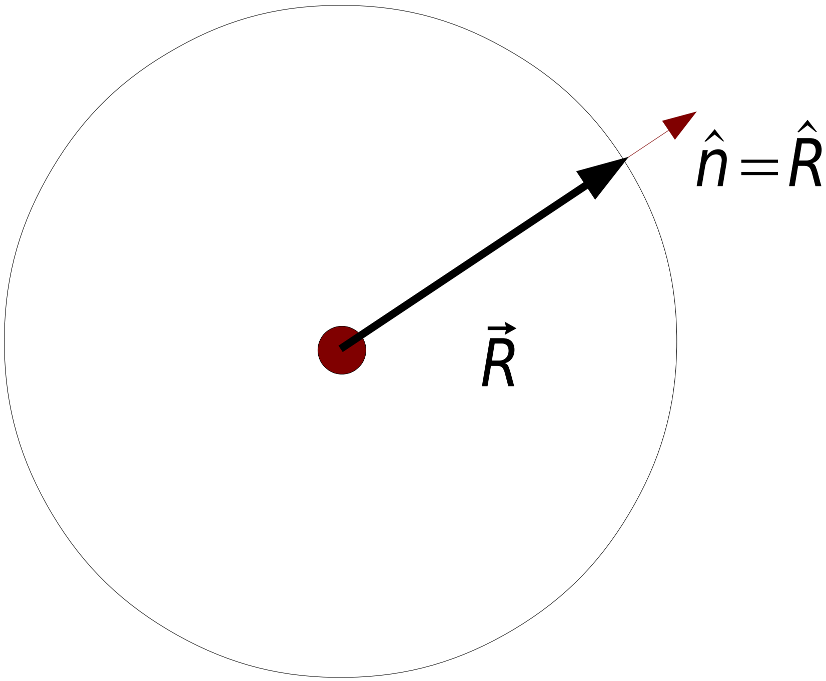
do integrals In class!

$$\boxed{\oint dA = 4 \pi R^2}$$

Next: construct the area element *dA*

Electric Flux Through A Closed Surface

Lets consider the flux through the surface of a sphere (radius R) that contains a charge q at the centre.



$$\Phi_E = \frac{k_e q}{R^2} 4\pi R^2$$

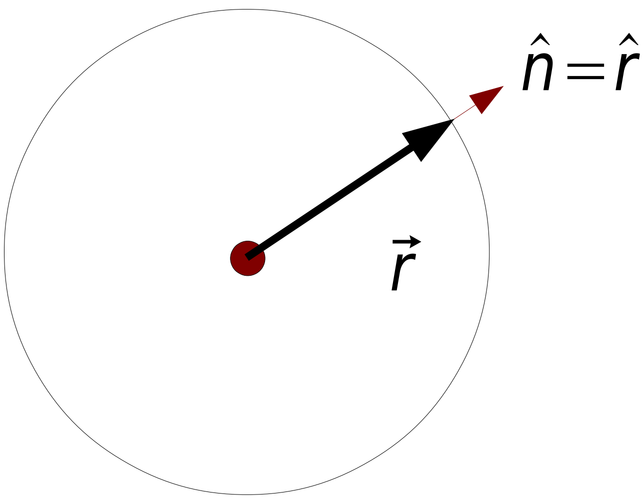
$$= \frac{1}{4\pi\epsilon_0} q(4\pi) = \frac{q}{\epsilon_0}$$

The total flux out of the closed spherical surface is simply the total charge enclosed divided by a constant.

$$\Phi_E = \frac{q}{\epsilon_0}$$

Electric Flux Through A Closed Surface

Make the sphere radius smaller (call it $\vec{r}$) and NOTHING CHANGES



$$\Phi_E = \frac{k_e q}{r^2} 4\pi r^2$$

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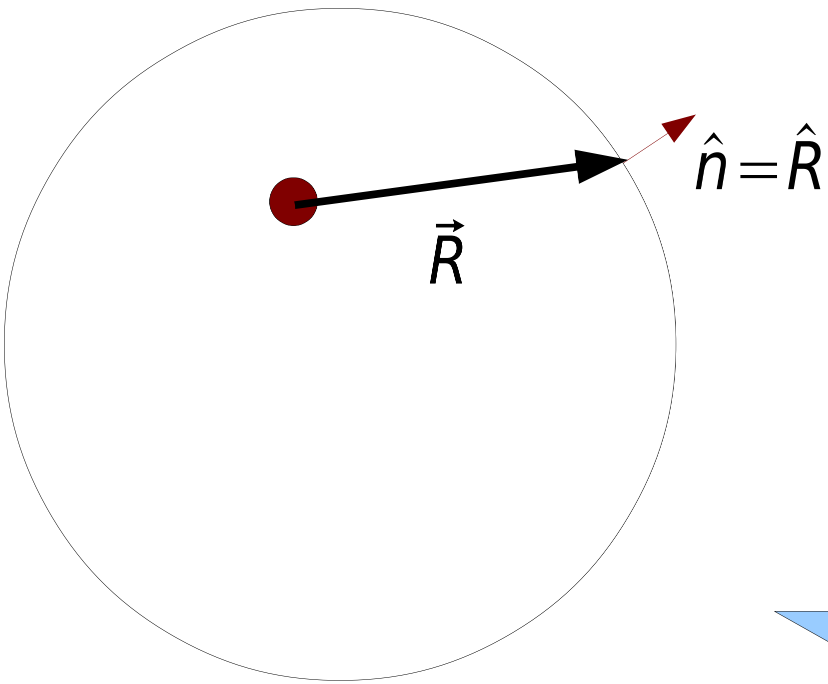
The total flux out of the closed spherical surface is simply the total charge enclosed divided by a constant.

$$\Phi_E = \frac{q}{\epsilon_0}$$

Electric Flux Through A Closed Surface: Gauss' Law

Now here's the really remarkable thing. No matter where I place the charge... or how many charges I place inside the sphere...



Even though the field at each individual point on the surface will be different...

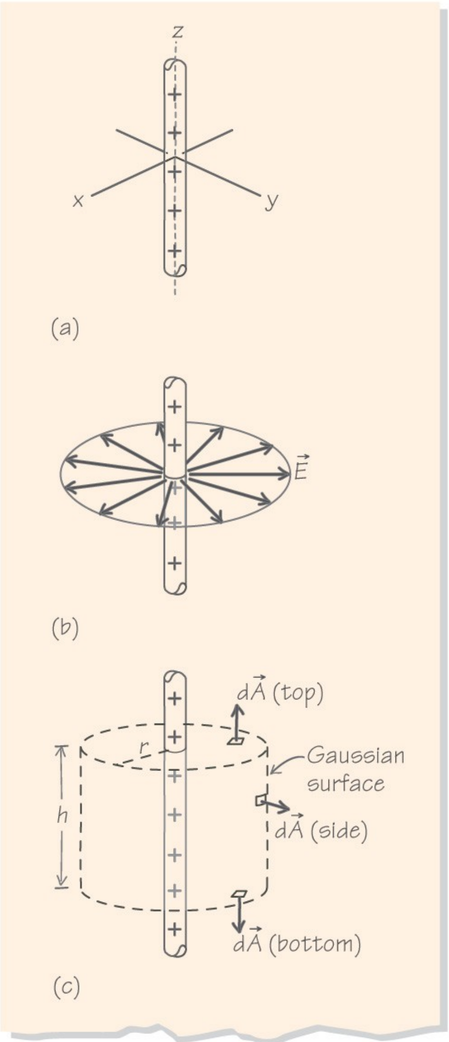
The total flux out of the closed spherical surface is STILL simply the total charge enclosed divided by a constant.

$$\Phi_E = \frac{q}{\epsilon_0}$$

Gauss' Law: Examples

A Line of Charge

Use Gauss' Law to find the electric field a distance r from a line of charge that extends to infinity and has a charge per unit length of λ



do in class

The Electric Field due to a continuous charge distribution

The General Case. Coulombs law for a distribution of charges:

$$\Delta \vec{E} = k_e \frac{\Delta q_i}{r_i^2} \hat{r}_i \qquad \text{and} \qquad \vec{E} = \sum_i \Delta \vec{E} = k_e \sum_i \frac{\Delta q_i}{r_i^2} \hat{r}_i$$

Take the limit as the element of charge are infinitesimal

$$d\vec{E} = k_e \frac{dq}{r^2} \hat{r}$$

$$\vec{E} = k_e \int \frac{dq}{r^2} \hat{r}$$

The Electric Field due to a continuous distribution of charge

Volume Charge Density. *If a charge Q is uniformly distributed throughout a volume V, then we may define a volume charge density ("rho"):*

$$\rho = \frac{Q}{V}$$

Surface Charge Density. *If a charge Q is uniformly distributed throughout a surface area A, then we may define a surface charge density ("sigma"):*

$$\sigma = \frac{Q}{A}$$

Linear Charge Density. *If a charge Q is uniformly distributed throughout a line of length l, then we may define a linear charge density ("lambda"):*

$$\lambda = \frac{Q}{l}$$

The Electric Field due to a continuous distribution of charge

Depending on whether the charge is distributed in the entire volume, or the surface or just a line....

Volume Charge Density. $\rho = \dfrac{Q}{V}$ ⇒ $dq = \rho \, dV$

Surface Charge Density. $\sigma = \dfrac{Q}{A}$ ⇒ $dq = \sigma \, dA$

Linear Charge Density. $\lambda = \dfrac{Q}{l}$ ⇒ $dq = \lambda \, dl$

Then the integral: $\vec{E} = k_e \int \dfrac{dq}{r^2} \hat{r}$ **is well-defined, because**

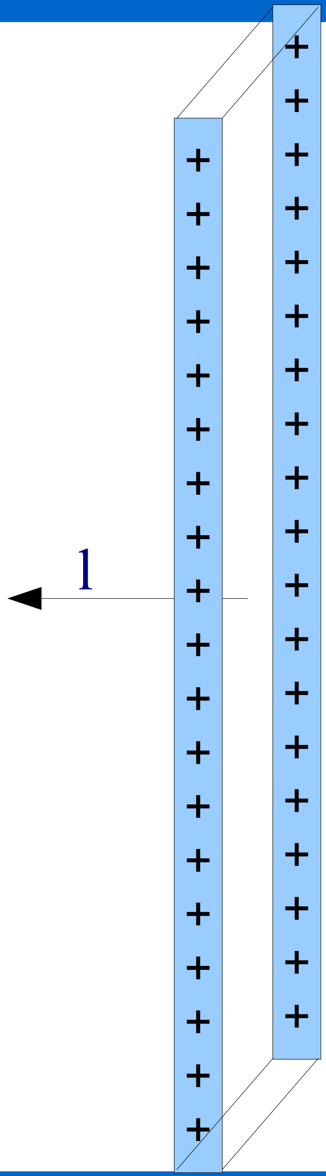
$\rho \, , \sigma \, , \lambda$ **are constants**

Gauss' Law: Examples

A Nonconducting Plane Sheet of Charge

Use Gauss' Law to find the electric field a distance l from a non-conducting infinite plane with charge per unit area σ

Note that innocuous word "non-conducting"
We will deal with conductors (e.g. metals) next week.



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