We present a detailed analysis of the magnetic states of monoclinic multiferroic systems as a function of temperature and magnetic field. The proposed non-local Landau free energy derived using symmetry arguments accounts well for the phase diagram of monoclinic MnWO₄ with the field applied along each of the three principal crystallographic directions. A description is proposed for the high field magnetic spin configurations, above 10 T, which have not yet been identified by neutron scattering. The nature of the induced electric polarization in each of the magnetic states is described. Corresponding results for CuO are also presented.

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I. INTRODUCTION

A common feature of a large number of multiferroics is ferroelectricity induced by a spiral magnetic order associated with magnetic frustration. Noncollinear magnetic order breaks the inversion symmetry giving rise to magnetoelectric coupling and the coexistence of magnetic and ferroelectric order. In some cases this can occur at high temperatures and therefore this class of magnetically frustrated multiferroics offers new opportunities for technological applications.

The interest in this field has grown considerably after the discovery of new materials such as rare-earth (R) manganites RMnO₃, RMn₂O₅, geometriyally frustrated systems CuMO₂ (M is Fe or Cr), and monoclinic multiferroic compounds like CuO and MnWO₄. Even though these materials can belong to a different point group in the paramagnetic state, the magnetoelectric effect is regularly observed in the monoclinic structural phase, which is the case for CuMO₂, CuO, and MnWO₄. Elucidating the nature of the often complex spin structures can serve to further our understanding of how ferroelectric and magnetic order are linked in monoclinic systems and multiferroics in general. Among all these materials, MnWO₄ has been the subject of many experimental and theoretical studies. In particular, the magnetic phase diagram of MnWO₄ has been determined up to 60 T with the field applied along the three principal crystallographic directions. Up to seven magnetic states have been determined. Moreover, polarization measurements have identified three states for which the magnetic and ferroelectric order coexist. Thus, this extensive set of experimental data offers a unique opportunity to test theoretical models for monoclinic multiferroics in general.

Encouraged by our recent successes on modeling the magnetic phase diagram of CuO we extend here this development of a nonlocal Landau-type free energy to study a wider class of monoclinic multiferroics. This model, which takes into consideration the crystalline anisotropy and the magnetoelectric coupling, successfully accounts for the magnetic phase diagrams of MnWO₄ for different field directions. The model is also used to identify the high-field spin configuration of magnetic orders not yet determined by neutron scattering measurements. Features of the spiral orders leading to a ferroelectric polarization are also determined and an application of the model is made to CuO.

In Sec. II we provide a short description of the symmetry, structural, magnetic properties, and ferroelectric properties of MnWO₄ relevant to the model. In Sec. III, details regarding the derivation of the Landau free energy and the magnetoelectric coupling are given. The magnetic phase diagrams, for different field directions, are presented in Sec. IV using a set of numerical parameters determined using the main characteristics of MnWO₄. The nature of the spin configurations associated with the different magnetic orders, as well as results regarding the electric polarization, are presented in Sec. IV A - IV D. In Sec V, we present the magnetic phase diagram of CuO using the same model. Finally, the principal results are summarized and discussed in Sec. VI.

II. PROPERTIES OF MNWO₄

Like other magnetoelectric compounds (CuFeO₂, CuCrO₂, and CuO), the low temperature point group of MnWO₄ is monoclinic (space group P21/c). Furthermore, this particular multiferroic can be viewed as zigzag-chains along the c-axis (see Ref. [14]) with two Mn^{2+} magnetic ions per unit cell (S = 5/2), located at τ₁ = (0.5, y, 0.25) and τ₂ = (0.5, 1 - y, 0.75) with y = 0.685. According to neutron scattering experiments the magnetization easy-axis, in the ac-plane, is at an angle of 37° from the monoclinic a-axis, while the two-fold symmetry axis is along the b-axis. As with CuO, MnWO₄ exhibits three antiferromagnetic phases (AF1, AF2, and AF3) at zero field. The low temperature phase, AF1 (T_{N1} = 7.4 K), corresponds to a commensurate collinear spin order with k = (-1/4, 1/2, 1/2) with the moments pointing along the easy-axis. The intermediate phase, AF2...
(T_{N2} = 12.5 \, \text{K}), is a spiral state with moments in a plane defined by the easy-axis and the b-axis. Finally, the high temperature phase AF3 (T_{N3} = 13.5 \, \text{K}) is associated with an incommensurate sinusoidal modulation with moments only ordered along the easy-axis. Both incommensurate phases share the same wave vector \mathbf{k} = (-0.214, 1/2, 0.457).

Recently, sound velocity\textsuperscript{15} and electric polarization\textsuperscript{19} measurements have been used to extend the magnetic phase diagram of MnWO\(_4\) up to 60 T. According to these results, three additional phases are detected (HF, IV, and V) with the magnetic field parallel to the easy-axis, while two new phases (X and V') are induced with the field along the b-axis. It has also been shown that phase IV sustains a ferroelectric polarization along the b-axis (as in AF2), while the polarization switches to be along the a-axis in phase X.\textsuperscript{10,19} Considering that polarized neutron diffraction measurements in MnWO\(_4\)\textsuperscript{11,21} show a clear correlation between the induced ferroelectric polarization and the AF2 chiral magnetic order, these recent results suggest that the new phases IV and X might be associated with a chiral magnetic order. Since no neutron scattering data exist above 10 T, the exact spin configuration of the field induced magnetic orders remain unknown. Analysis of the proposed non-local Landau-type free energy helps to elucidate the nature of the AFM states observed in MnWO\(_4\) and possibly in other monoclinic multiferroics.

### III. THE MODEL

The derivation presented here is essentially based on the formulation of a non-local Landau free-energy which is known to successfully accounts for the phase diagram and properties of triangular antiferromagnets\textsuperscript{22-25} as well as some multiferroics (CuFeO\(_2\) and CuO).\textsuperscript{20,26} As in previous contributions, the integral form of the free energy \(F_s\) is represented by a Taylor expansion in terms of the spin density \(s(\mathbf{r})\):

\[
F_s = \frac{1}{2V} \int d\mathbf{r}_1d\mathbf{r}_2 A_{ij}(\mathbf{r}_1 - \mathbf{r}_2)s_i(\mathbf{r}_1)s_j(\mathbf{r}_2)
+ \frac{1}{4V} \int d\mathbf{r}_1d\mathbf{r}_2d\mathbf{r}_3d\mathbf{r}_4 B_{ijkl}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4)s_i(\mathbf{r}_1)s_j(\mathbf{r}_2)s_k(\mathbf{r}_3)s_l(\mathbf{r}_4) - \int s(\mathbf{r})\cdot \mathbf{H} \, d\mathbf{r},
\]

where repeated indices represent summations relative to the coordinates \(i, j, k, l = a, b, c'\). For convenience, we define the spin density using a cartesian coordinate system such that \(a \perp b \perp c'\), where \(a\) and \(b\) coincide with the monoclinic crystal axes (the two fold monoclinic symmetry axis being the \(b\)-direction), while \(c'\) is almost parallel to \(c\). For simplification, the expansion is truncated to 4-order terms with the addition of a Zeeman energy term (last term of Eq. 1). All independent contributions to \(F_s\) are then evaluated using a magnetic moment density \(s(\mathbf{r})\) defined in term of a spin density \(\rho(\mathbf{r})\) such that

\[
s(\mathbf{r}) = \frac{V}{N} \sum_R \rho(\mathbf{r})\delta(\mathbf{r} - \mathbf{R}),
\]

with \(\mathbf{R}\) representing the lattice coordinates of the magnetic Mn\(^{2+}\) ions (\(S = 5/2\)). In conformity with experimental observations,\textsuperscript{12} we assume that the spin density is modulated by a single wave vector \(\mathbf{Q}\) describing the long-range magnetic order such that

\[
\rho(\mathbf{r}) = \mathbf{m} + \mathbf{S} e^{i\mathbf{Q} \cdot \mathbf{r}} + \mathbf{S}^* e^{-i\mathbf{Q} \cdot \mathbf{r}},
\]

where \(\mathbf{S}\) is the spin polarization vector and \(\mathbf{m}\) is the induced magnetization.

The free energy must be invariant with respect to the point group operations of the corresponding space group P2/c. This includes a two fold symmetry operation along the \(b\)-axis from which we identify four second order invariants \(A_{ij}\). These contributions are written in term of the usual temperature dependent isotropic term \(A_0\mathbf{S}\cdot \mathbf{S}^* = a(T+T_Q)\mathbf{S}\cdot \mathbf{S}^*\) and three anisotropic terms. In order to focus on the most relevant terms, only anisotropy terms in \(\mathbf{S}\) at fourth order are included. After integrating over the spatial degrees of freedom, the free energy can be expressed as

\[
F_s = A_Q \mathbf{S} \cdot \mathbf{S}^* + A_{aa}S_aS_a^* + A_{bb}S_bS_b^* + \frac{A_{cc'}}{2}(S_cS_c^* + S_{a'}S_{a'}^*) + B_1(\mathbf{S} \cdot \mathbf{S}^*)^2 + \frac{B_2}{2}\mathbf{S} \cdot \mathbf{S} \]
\[
+ \frac{B_4}{4} \left((\mathbf{S} \cdot \mathbf{S})^2 + (\mathbf{S}^* \cdot \mathbf{S}^*)^2\right) \Delta_{A_Q,G} + \frac{1}{2}A_m m^2 + \frac{1}{4}B_3m^4 + 2B_4\mathbf{m} \cdot \mathbf{S}^2 + B_5m^2\mathbf{S} \cdot \mathbf{S}^* - \mathbf{m} \cdot \mathbf{H},
\]
where $A_o = a(T - T_o)$. Note that the lattice periodicity allows for an umklapp term $\Delta_{Q,G}$. This particular term leads to first order transitions and accounts for the period-4 commensurate states observed at low temperatures in CuO, CuFeO$_2$, and MnWO$_4$. In order to account for non-collinear magnetic configurations the complex spin polarization vector is expressed in the general form

$$S = S_1 + iS_2.$$  \hspace{1cm} (5)

To account for the electric polarization, due to chiral states, terms associated with the induced polarization $P$ due to the magnetoelectric coupling are included in the free energy $F_E$. For magnetoelectric monolayer systems this gives low-order contributions

$$F_E = \frac{1}{2} \chi_a \left( \frac{1}{2} P_a^2 + \frac{1}{2} P_b^2 + \frac{1}{2} P_{c'}^2 \right) + F_{ME},$$  \hspace{1cm} (6)

where $\chi_i$ ($i = a, b, c'$) represents the dielectric susceptibility of the paraelectric phase along the principal axes ($\chi_{c'} \sim \chi_c$). For the present analysis, it is assumed that magnetoelectric coupling energy $F_{ME}$ is due to the inverse Dzyaloshinskii-Moriya (DM) or the spin-current model as they account for the experimental results in this class of systems. The magnetoelectric energy can be then expressed as

$$F_{ME} = \left( \frac{1}{2V} \int d\mathbf{r}_1 d\mathbf{r}_2 \alpha(\mathbf{r}) \mathbf{\tau} \times [S(\mathbf{r}_1) \times S(\mathbf{r}_2)] \right) \cdot P,$$  \hspace{1cm} (7)

which depends on the double cross product between the spin density $S(\mathbf{r}_1), S(\mathbf{r}_2)$, and the vector $\mathbf{\tau} = \mathbf{r}_1 - \mathbf{r}_2$ connecting both moments, with $\alpha(\mathbf{r})$ being a coupling coefficient. For a spin modulation with a wave vector $Q$, the magnetoelectric energy can be more conveniently written as a function of the spin polarization vectors $S_1$ and $S_2$ as above.

Considering that MnWO$_4$ can be treated as a set of zigzag-chain parallel to the $c$-axis, with magnetic nearest neighbors located at $\mathbf{r}_1 = (0.5, y, 0.25)$ and $\mathbf{r}_2 = (0.5, 1 - y, 0.75)$ with $y = 0.685$, the magnetoelectric energy reduced to

$$F_{ME} = -\alpha \sin(2\pi y) \left[ (S_{1a} S_{2c'} - S_{1c'} S_{2a}) P_a + (S_{1b} S_{2c'} - S_{1c'} S_{2b}) P_b \right].$$  \hspace{1cm} (8)

This result immediate indicates that the mechanism considered here can account for a spin-driven electric polarization pointing in the $ab$-plane, consistent with the experimental observations. Also note that this result shows that the zigzag structure of the chains ($y \neq 0$) plays a crucial role in the existence of an electric polarization in MnWO$_4$.

### IV. MAGNETIC PHASE DIAGRAM OF MnWO$_4$

A number of Landau-type models have been proposed to account for the magnetic properties of MnWO$_4$. However, in general these consider only one or two of the anisotropic contributions. As shown below, all of the allowed second-order anisotropic terms ($A_{ij}$) play a crucial role in the determination of the spin states and phase diagrams of MnWO$_4$. Here, the relative values of these coefficients are set using a few key experimental results. Considering that the easy-axis is in the $ac$-plane, with a secondary easy-axis is along $b$, as determined by neutron scattering, we impose that $|A_{ac'}| > |A_{bb}| > |A_{aa}|$. Moreover, minimization of Eq. 1 indicates that the direction of the easy-axis ($\gamma = 37^\circ$) is given by $\tan(2\gamma) = A_{ac'}/A_{aa}$. Thus, in order to reproduce the phase sequence for $0 < T < 1.2$, we set $a = 1, T_Q = 1.18, T_o = -1.0, A_{ac'} = -0.03$, and $A_{bb} = -0.0115$, and $A_{aa} = -0.011$. All other coefficients are determined using analytical and numerical results associated with a set of conveniently defined critical values such that $T_{N3} = 1.2, T_{N2} = 1.11, H_c(T = 1.15) = 1.35$, and a maximum critical field $H_c(T = 0) = 6.0$. Therefore, $B_1 = 0.103, B_2 = 0.011, B_3 = 0.0633, B_4 = 0.0125$, while we set $B_5 = 0.1$ for convenience. Considering that the low temperature phase (AF1) is attributed to the period-4 umklapp term, the coefficients $B_u = 0.0206$ and $T_{Q = G/A} = 1.15$ have been used to reproduce a first order phase transition at $T_{N1} = 0.19$. Finally, the numerical values used for the dielectric susceptibility have been set based on experimental results such that $\chi_a = 15.3, \chi_b = 12.2$, and $\chi_{c'} = 11.0$, with a magnetoelectric coupling coefficient arbitrary set to $\zeta = 0.015$.

With all parameters set, the magnetic phase of MnWO$_4$ has been determined by numerical minimization of the total free energy $F = F_o + F_E$ for different field directions, without adjusting any parameters. For an easy comparison with published experimental data on MnWO$_4$, we present in Fig. 1 the magnetic phase diagrams obtained for a field applied along the easy-axis ($\hat{x}$), a field parallel to the $b$-axis ($\hat{y}$), and for a field simultaneously normal to the easy and $b$-axis ($\hat{z}$). For clarity, we also identify the different phases using the labeling sequence found in Ref. 19.

At zero field the model predictions agree with cor-
In general, the qualitative agreement between the numerical predictions and the experimental data, for different field directions, is remarkable. For example, with the field applied along the b-axis (\(\hat{y}\)), the field dependence of the boundary associated with the AF1 state emerges spontaneously. A more detailed discussion of the phase diagram associated with each field direction is presented below.

### A. \(H \parallel \) easy-axis (\(\hat{x}\))

As the field along the easy-axis (\(\hat{x}\)) increases, numerical calculations at \(T = 0\) show a first-order phase transition between the AF1 to the AF2 state. Here, spins antiparallel to the field flip to the \(\hat{b}\)-direction, leading to the AF2 incommensurate chiral order also observed at zero field. At higher fields, moments still parallel to the field (easy-axis) suddenly flip to the \(\hat{b}\)-axis. This first-order transformation gives rise to a new period-4 commensurate state HF with the moments pointing along the \(\hat{b}\)-axis. With further increase in field, the phase HF is replaced by a new chiral configuration (IV) which induces an electric polarization pointing along the \(\hat{b}\)-axis (\(\hat{y}\)). Finally, our model indicates that phase V corresponds to a sinusoidal incommensurate configuration with moments only aligned along the \(\hat{b}\)-axis (\(\hat{y}\)). Here, this configuration minimizes the energy as the moments are normal to the field. The \(\hat{b}\)-axis is favored as it corresponds to the intermediate easy-axis. A description of the magnetic phases is illustrated in Fig. 2 where the spin polarization vectors are given relative to the easy-axis (\(\hat{x}\)), the intermediate easy-axis (\(\hat{y}\)), and the hard-axis (\(\hat{z}\)).

Fig. 3 shows the field dependence of the polarization, \(P_b\), determined at \(T = 0\) and \(T = 0.6\) with the magnetic field along \(\hat{x}\). As AF1, HF, and phase V correspond to collinear magnetic states, the electric polarization vanishes. At low temperatures, the period-4 collinear phase (AF1) is stabilized with the moments along the easy-axis (\(\hat{x}\)). In the intermediate temperature range, a spiral order with the moments along the easy-axis (\(\hat{x}\)) occurs (AF2). Finally, at higher temperatures a collinear sinusoidal order emerges with spins ordered only along the easy magnetization direction (\(\hat{x}\)). Furthermore, minimization of the free energy with respect to the polarization shows that the chiral order (AF2) induces an electric polarization pointing along the \(\hat{b}\)-axis, as determined experimentally.\(^{11,21}\)

With an applied field along the each of the three axes (see Fig. 1), the analysis of the model reveals seven different phases, consistent with the experimental observations. In general, the qualitative agreement between the corresponding neutron scattering data.\(^{12}\)
ishes, while the magnetic chiral states (AF2 and IV), support a polarization along the b-axis. Numerical calculations performed at $T = 0.6$ also indicate that, due to the magnetoelectric coupling (Eq. 6), magnetic moments initially parallel to the easy-axis ($\hat{x}$) gradually rotate toward the hard-axis direction ($\hat{z}$) as $H$ increases. This observation is supported by additional simulations which omitted the coupling term and in that case the moment suddenly flip from the $\hat{x}$ direction to $\hat{z}$. Thus, as shown in Fig. 3, the rotation of the chiral plane results in a maximum on the polarization $P_b$, which coincides with the AF2 - IV phase boundary. However, note that the experimental results rather show a change in the sign of the polarity of $P_b$ between AF2 and IV. We believed that this behavior can be attributed to higher order terms not considered in the present model. Considering the large number of possible 6-order invariant terms, it would be cumbersome to carry out a detailed analysis of each contributions. Nevertheless, we present in the inset of Fig. 3 the effect of including the higher magnetoelectric coupling term $\zeta \mathbf{P} \cdot \mathbf{P}_{\text{ME}} m^2$. In that case, the model does account for the change in sign of the polarity. Considering that the polarization is given by $P_b \propto S_{1b}S_{2b} \propto \sin \gamma$ (Eq. 8), where $\gamma$ is the angle that the moments make in the $ac'$-plane relative to the a-axis, we conclude that the chiral plane must be normal to the $c'$-axis in order to switch the polarity of the polarization $P_b$.

**B. $\mathbf{H} // b$-axis ($\hat{y}$)**

With the field applied along the $b$-axis, a new phase X is stabilized above the chiral phase AF2 as shown in Fig. 1b. According to our calculations, this phase results from the flip of $\mathbf{S}$ from $b$ to the hard-axis ($\hat{z}$). As shown in Fig. 2, the resulting phase is therefore a new chiral ordered state with the moments in the $ac$-plane (equivalent to the $xz$-plane). Thus, the polarization switches to the $a$-direction (see Eq. 8). This is illustrated in Fig. 4 which shows the numerical predictions for both polarizations for a field scan crossing the AF2-X phase boundary ($T = 0.6$). These predictions agree well with experimental observations. Naturally, as phases AF3 and AF1 correspond to collinear orders no polarization is detected.

**C. $\mathbf{H} // hard$-axis ($\hat{z}$)**

For a field applied along the hard-axis, no new magnetic order is obtained, as shown in Fig. 1c. This is to be expected as all moments lay in the $xy$-plane at zero field. The application of a field along $\hat{z}$ only results in the canting of the moments out of that plane, inducing a magnetization along the field direction.

**D. $\mathbf{H} // (\hat{x} + \hat{y})$**

To complete our comparison between model and experimental results for the magnetic properties of MnWO$_4$, we present in Fig. 5 the magnetic phase diagram for a field $45^\circ$ away from the easy-direction ($\hat{x}$) in the $xy$-plane. At low temperatures, we find that the magnetic order corresponds to a commensurate spin configuration ($\uparrow\uparrow\downarrow\downarrow$) with the moments gradually rotating from the easy-axis toward the field direction as the field is increased. Considering that the moments remain in the $xy$-plane, this state can be described as a superposition of the collinear phases AF1 and HF. As the moments become parallel to the field direction, we observe a first-order transition to a chiral state corresponding to the
superposition of the IV and X state. These observations are supported by the calculated field dependence of the polarizations shown in Fig. 6 for $T = 0$ and 0.6. In the IV+X phase, we find that the polarization has components along both $a$ and $b$ directions. Moreover, due to the tilt of the moments within the chiral plane, a small polarization along the $a$-axis is obtained in the AF2$'$ state.

V. MAGNETIC PHASE DIAGRAM OF CUO

The proposed model is also relevant for other monoclinic ferroelectrics. This can be illustrated by applying it to examine the properties of the high temperature multiferroic compound CuO (also see Ref. 20), which shows the same sequence of magnetic ordering as in MnWO$_4$ at zero field. However, in this case the easy-axis is the $b$-axis, while the intermediate easy-axis is in the $ac$-plane. Therefore, in order to properly account for the anisotropic properties of CuO, we set $A_y = -0.02$, $A_{xy} = -0.01$, and $A_x = 0.012$ with $B_{a} = 0.02464$, $C_x = 2.0$, $C_y = 3.0$, $C_z = 2.5$, and $\zeta = 0.027$, while the other coefficient values remain unchanged. Moreover, CuO is composed of Cu ion sheets formed by two set of chains with ferro and antiferromagnetic coupling, respectively. In this case the magnetoelectric energy reduces to

$$F_{ME} = -2\alpha \left[(S_{1a}S_{2c} - S_{1c}S_{2a})P_a + ((S_{1a} - S_{1c})S_{2b} + S_{1b}(S_{2c} - S_{2a}))P_b + (S_{1a}S_{2c} - S_{1c}S_{2a})P_c\right].$$

(9)

As the AF2 chiral state has $S_1 = S \hat{b}$, the magnetoelectric coupling leads to a polarization along the $b$-axis. As the $b$-axis is the easy-axis, the magnetic phase diagram for CuO derived for a field parallel to the $b$-axis (Fig. 7) can be compared to the one obtained for MnWO$_4$ with the field along the $\hat{x}$-direction (see Fig. 1). We notice that, for $T > 0.3$, the results derived for CuO are very similar to those obtained for MnWO$_4$. We obtain the magnetic orders illustrated in Fig. 2 in terms of the easy-axis ($\hat{x}$), the intermediate easy-axis ($\hat{y}$), and the hard-axis ($\hat{z}$). Thus, the field induces phase HF1 is equivalent to HF, HF2 is IV, and AF3 is analog to V. So far, recent experimental ultrasonic velocity measurements on CuO have confirmed the existence of the phases AF1, AF2, AF3, and HF1. As well, note that the polarization associated with the chiral state HF2 should be pointing in the $ac$-plane according to Eq. 9. For a field applied along the intermediate easy-axis and along the hard-axis, the numerical magnetic field diagrams are similar to those presented in Fig. 1 for $H \parallel \hat{y}$ and $H \parallel \hat{z}$. Our calculations also indicate that the polarization remains along the $b$-axis between the AF2 and X chiral state. Therefore, additional measurements at higher fields ($H > 16$ T) are required to fully support the model’s predictions for CuO.

VI. SUMMARY AND CONCLUSIONS

This work has demonstrated that a comprehensive explanation of the complex magnetic-field – temperature

![Magnetic phase diagram of MnWO$_4$](image)

![Field dependence of the induced polarization](image)
phases are indicated by $P_a$ and $P_{ac}$.

The predicted electric polarization induced by non-collinear spin configurations are described and agree with experimental results. Corresponding results for CuO with the field applied along the easy axis are also included and agree well with the observed phase diagram in that case. The model predicts that for the field applied along the other axes, the phase diagrams should be similar to MnWO$_4$.

Future work will include a study of the doped compound Mn$_{1-x}$Co$_x$WO$_4$ where recent neutron diffraction results have revealed a complex phase diagram as a function of $x$ and temperature. Substitution of Mn with Co modifies the local magnetocrystalline anisotropy and can be described in our model through variations in the second-order coefficients $A_{ij}$. There is also evidence for a phase (AF4 + AF5) where the spin configuration is described by more than one wavevector which can be accounted for by adding a term to Eq. (3).

In summary, the results presented here, in conjunction with previous work on CuFeO$_2$, show that symmetry considerations applied to a simple mean-field model which includes the inverse DM magnetoelectric coupling term provides a powerful tool for the description of magnetic and electric polarization ordering for a wide class of magnetoelectric compounds.

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