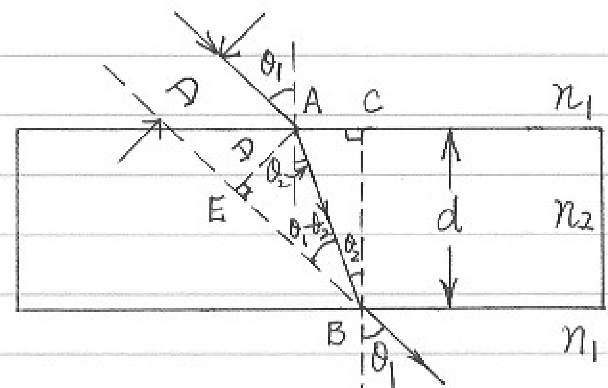
Physics 3600 Optics and Photonics I
Winter 2006
Assignment #1 Solutions

1.



In the figure, draw two vertical lines to form two rectangular triangles ΔABC and ΔABE, Suppose the length of $\overline{AB} = x$.

From ΔABC: $\cos\theta_2 = \dfrac{\overline{CB}}{\overline{AB}} = \dfrac{d}{x} \Rightarrow x = \dfrac{d}{\cos\theta_2}$

From ΔABE: $\sin(\theta_1 - \theta_2) = \dfrac{\overline{AE}}{\overline{AB}} = \dfrac{D}{x} = \dfrac{D\cos\theta_2}{d}$

Considering $\sin(\theta_1 - \theta_2) = \sin\theta_1\cos\theta_2 - \sin\theta_2\cos\theta_1$

$$\therefore \sin\theta_1\cos\theta_2 - \sin\theta_2\cos\theta_1 = \frac{D\cos\theta_2}{d}$$

$$D = \frac{d}{\cos\theta_2}(\sin\theta_1\cos\theta_2 - \sin\theta_2\cos\theta_1) = d\left(\sin\theta_1 - \frac{\sin\theta_2\cos\theta_1}{\cos\theta_2}\right)$$

From Snell's Law, $n_1\sin\theta_1 = n_2\sin\theta_2$

Using the relation: $\sin^2\theta + \cos^2\theta = 1$

$$\therefore D = d\left(\sin\theta_1 - \frac{n_1\sin\theta_1\cos\theta_1}{n_2\cos\theta_2}\right)$$

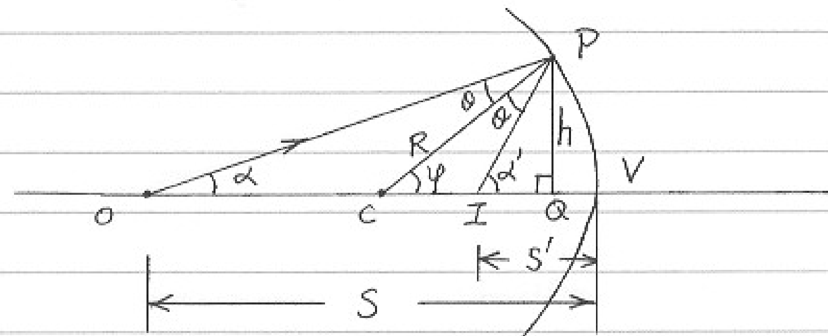
$$= d\sin\theta_1\left(1 - \frac{n_1\cos\theta_1}{n_2\cos\theta_2}\right)$$

$$= d\sin\theta_1\left(1 - \frac{n_1\cos\theta_1}{n_2\sqrt{1-\sin^2\theta_2}}\right)$$

$$= d\sin\theta_1\left(1 - \frac{n_1\cos\theta_1}{n_2\sqrt{1-\left(\frac{n_1}{n_2}\right)^2\sin^2\theta_1}}\right)$$

$$= d\sin\theta_1\left(1 - \frac{n_1\cos\theta_1}{\sqrt{n_2^2 - n_1^2\sin^2\theta_1}}\right)$$

2.



As we know, an exterior angle of an triangles = Sum of interior and opposite angles.

From ΔOPC : $\varphi = \alpha + \theta$ (1)

From ΔOPI : $\alpha' = 2\theta + \alpha$ (2)

(1) × 2 − (2) :

$$2\varphi - \alpha' = \alpha$$

$$\therefore \quad \alpha + \alpha' = 2\varphi \quad (3)$$

Applying the small angle (paraxial) approximation

$$\alpha \cong \tan\alpha = \frac{h}{S}$$

$$\alpha' \cong \tan\alpha' = \frac{h}{S'}$$

$$\varphi \cong \tan\varphi = \frac{h}{R},$$ Since VQ is small when φ is small

$$\therefore \quad \frac{h}{S} + \frac{h}{S'} = \frac{2h}{R}$$

$$\Rightarrow \frac{1}{S} + \frac{1}{S'} = \frac{2}{R}$$

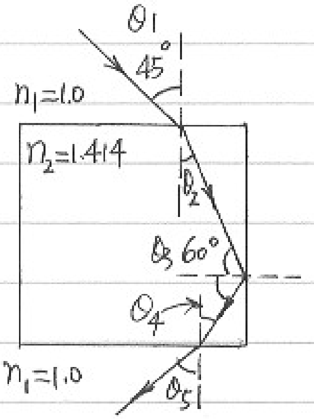
Note:

(1) For a convex mirror, the mirror equation is $\frac{1}{S} - \frac{1}{S'} = -\frac{2}{R}$

See class notes or See textbook Pedrotti, pages 43−45.

(2) With the appropriate sign convention, all possibilities can be represented by the equation $\frac{1}{S} + \frac{1}{S'} = -\frac{2}{R}$

3.



At top: $1.0 \times \sin 45° = \sqrt{2} \cdot \sin \theta_2$

$\therefore\ \theta_2 = 30°$

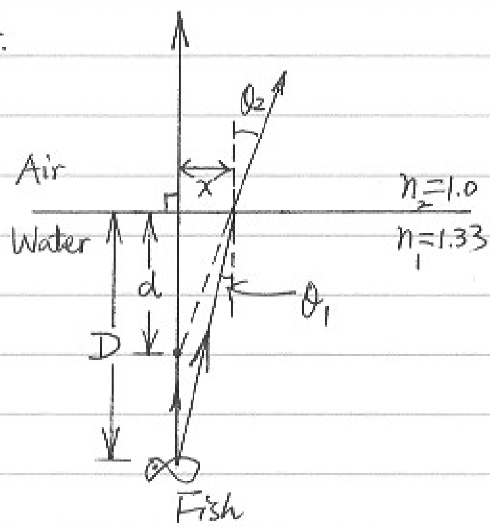
At side: $\sqrt{2}\, \sin 60° = 1.0 \times \sin \theta'$

$\sin\theta' = \sqrt{1.5} > 1$

Thus total internal reflection occurs

At bottom: reverse of top: $\theta_5 = 45°$

4.



Look at two rays coming from fish: one is normal to surface. The other one hits the interface at a small angle θ_1, and is transmitted at θ_2.

If we project the transmitted ray at θ_2 back to the normally incident ray, we get the apparent location of the fish (d)

Snell's Law (small angle): $\theta_2 \cong n_1\theta_1$.

While $\dfrac{x}{d} \cong \theta_2$, $\dfrac{x}{D} = \theta_1$

$\therefore\ \theta_2 d = \theta_1 D$

$\Rightarrow\ n_1\theta_1 d = \theta_1 D$

$\Rightarrow\ d = \dfrac{D}{n_1} = \dfrac{1\,m}{1.33} = 0.75\,m$

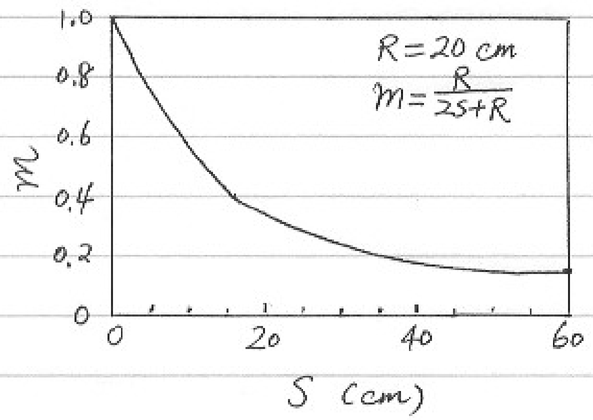
5. Mirror equation $\frac{1}{s} + \frac{1}{s'} = -\frac{2}{R}$

$$\frac{1}{s'} = -\frac{2}{R} - \frac{1}{s}$$

$$\therefore \ s' = -\frac{1}{\frac{2}{R}+\frac{1}{s}} = -\frac{Rs}{2s+R}$$

$$m = -\frac{s'}{s} = \frac{R}{2s+R}$$

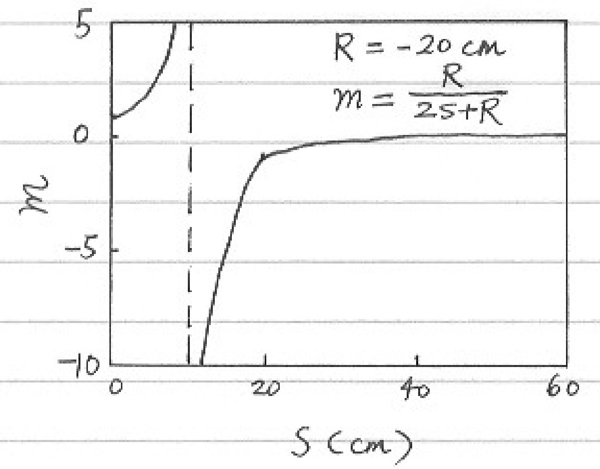
Since it is real object here ($s>0$), for positive R, m is always positive and s' is always negative.



Plot of m versus s for $R = +20$ cm.

For negative R, we will get a divergence in m for $2s = -R$, i.e., $s = f$, at focus $s' \to \infty$

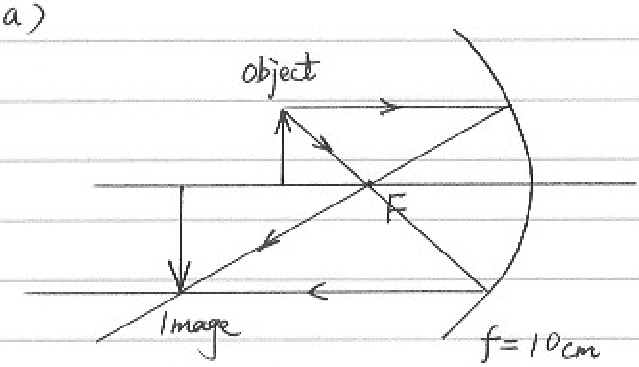
For $s>f$, s' is positive and m is negative (real image, inverted);
For $s<f$ (but positive), s' is negative and m is positive (virtual image, upright).



Plot of m versus s for $R = -20$ cm.

6.

(a)



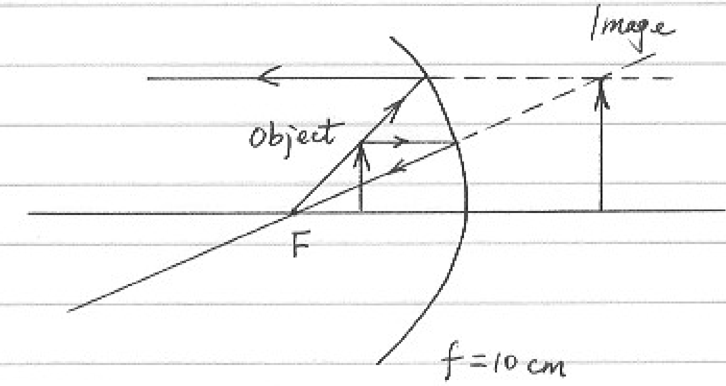
$$\frac{1}{S'} = \frac{1}{f} - \frac{1}{S} = \frac{1}{10} - \frac{1}{15} = \frac{3}{30} - \frac{2}{30} = \frac{1}{30}$$

$\therefore \quad S' = 30$ cm

$$m = -\frac{S'}{S} = -\frac{30}{15} = -2$$

Image is real.

(b)



$$\frac{1}{S'} = \frac{1}{f} - \frac{1}{S} = \frac{1}{10} - \frac{1}{5} = -\frac{1}{10}$$

$\therefore \quad S' = -10$ cm

$$m = -\frac{S'}{S} = -\frac{-10\,cm}{5\,cm} = 2$$

Image is virtual.