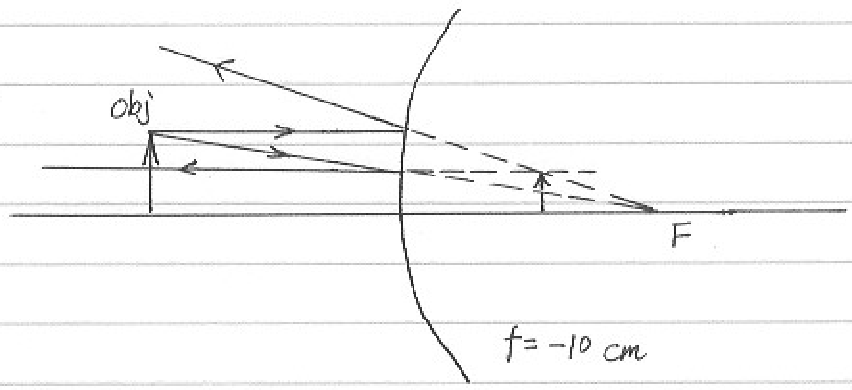
Physics 3600 Optics and Photonics I

Winter 2006

Assignment #2 Solutions

1.



$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = -\frac{1}{10} - \frac{1}{10} = -\frac{1}{5}, \quad \text{Hence } s' = -5 \text{ cm}$$

$$m = -\frac{s'}{s} = -\frac{-5}{10} = +0.5$$

Image is virtual.

2. For the refraction equation $\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R}$

Here, $\frac{n_1}{s} + \frac{n_2}{s'} = 0$

where, $n_1 = 1.33$, $n_2 = 1.0$, $R \to \infty$

$$\therefore \quad s' = -\frac{n_2}{n_1} s = -\frac{1.0}{\frac{4}{3}} \cdot 1.0 \text{ m} = -0.75 \text{ m}$$

– Negative sign tells us that the image is below the surface (virtual).

For magnification

$$m = -\frac{n_1 s_1'}{n_2 s_1} = -\frac{\frac{4}{3} \cdot (-\frac{3}{4})}{1.0 \times 1.0} = +1.0$$

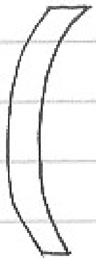
– We see the fish of its actual size.

3. From Lensmaker's equation $\frac{1}{s} + \frac{1}{s'} = \frac{n_2 - n_1}{n_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

It can be written as $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$, where $\frac{1}{f} = \frac{n_2 - n_1}{n_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Here, $n_2 = n = 1.5$, $n_1 = 1.0$, so $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

(a)

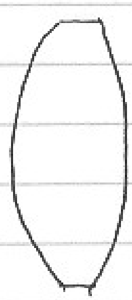


$$\frac{1}{f} = (n-1)\left(\frac{1}{10} - \frac{1}{10}\right) = 0$$
$$\therefore f = \infty$$

(b)

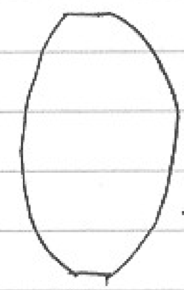
$$\frac{1}{f} = (n-1)\left(\frac{1}{10} - 0\right) = \frac{0.5}{10}$$
$$\therefore f = 20 \text{ cm}$$

(c)



$$\frac{1}{f} = (n-1)\left(\frac{1}{10} - \frac{1}{-20}\right) = 0.5 \cdot \frac{3}{20}$$
$$\therefore f = \frac{40}{3} = 13.3 \text{ cm}$$

(d)



$$\frac{1}{f} = (n-1)\left(\frac{1}{10} - \frac{1}{-5}\right) = 0.5 \cdot \frac{3}{10}$$
$$\therefore f = \frac{20}{3} = 6.67 \text{ cm}$$

(e)

$$\frac{1}{f} = (n-1)\left(\frac{1}{-10} - \frac{1}{-10}\right) = 0$$
$$\therefore f = \infty$$

(f)

$$\frac{1}{f} = (n-1)\left(\frac{1}{-10} - 0\right) = -\frac{0.5}{10}$$
$$\therefore f = -20 \text{ cm}$$

4. $\frac{1}{s'} = \frac{1}{f} - \frac{1}{s}$

(a) $\frac{1}{s'} = \frac{1}{10} - \frac{1}{100} = \frac{9}{100}$, $\therefore s' = \frac{100}{9} = 11.1$ cm

(b) $\frac{1}{s'} = \frac{1}{10} - \frac{1}{20} = \frac{1}{20}$, $\therefore s' = 20$ cm

(c) $\frac{1}{s'} = \frac{1}{10} - \frac{1}{10} = 0$, $\therefore s' = \infty$

(d) $\frac{1}{s'} = \frac{1}{10} - \frac{1}{5} = -\frac{1}{10}$, $\therefore s' = -10$ cm (virtual)

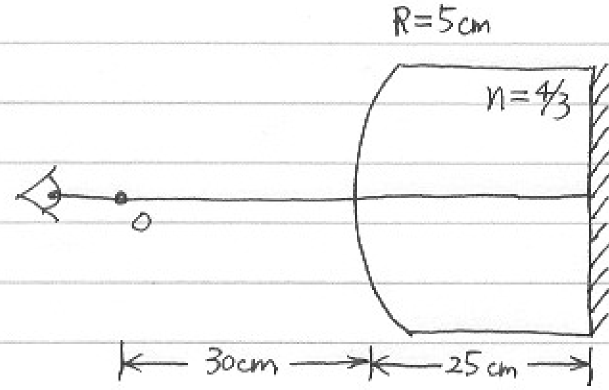
5. (a) $\frac{1}{s'} = -\frac{1}{10} - \frac{1}{100} = -\frac{11}{100}$, $\therefore s' = -9.09$ cm

(b) $\frac{1}{s'} = -\frac{1}{10} - \frac{1}{20} = -\frac{3}{20}$, $\therefore s' = -6.67$ cm

(c) $\frac{1}{s'} = -\frac{1}{10} - \frac{1}{10} = -\frac{1}{5}$, $\therefore s' = -5$ cm

(d) $\frac{1}{s'} = -\frac{1}{10} - \frac{1}{5} = -\frac{3}{10}$, $\therefore s' = -3.33$ cm

6.



Rays from the object undergo the following steps:

(a) refracted through the spherical window;

(b) then reflected from back plane mirror;

(c) then refracted out again through spherical window.

(a) $\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R} \Rightarrow \frac{1}{30} + \frac{4/3}{s'} = \frac{4/3 - 1}{5}$, $\therefore s' = 40$ cm

$m = -\frac{n_1 s'}{n_2 s} = -\frac{1 \times 40}{4/3 \times 30} = -1$

(b) $s = 25 - 40 = -15$ cm (virtual object)

$s' = -s = +15$ cm (from mirror equation $\frac{1}{s} + \frac{1}{s'} = \frac{-2}{R}$, where $R \to \infty$)

$m = -\frac{s'}{s} = -\frac{15\text{cm}}{-15\text{cm}} = +1$

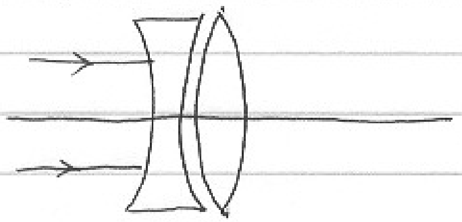
(c) $\frac{4/3}{10} + \frac{1}{s'} = \frac{1 - 4/3}{-5}$, $\therefore s' = -15$ cm

$m = -\frac{(4/3)(-15)}{(1)(10)} = +2.0$

Therefore, overall magnification $m = (-1)\cdot(+1)\cdot(+2) = -2$

Thus a virtual, inverted, double-sized image appears 15 cm behind (right) the spherical window.

7. (a) For cemented lenses with a configuration as



Applying lensmaker's equation to the negative lens for the parallel incident rays of light:

$$\frac{1}{s_1}+\frac{1}{s_1'}=\frac{1}{f_1} \Rightarrow 0+\frac{1}{s_1'}=\frac{1}{-5} \quad \therefore s_1'=-5 \text{ cm}$$

Taking the image formed by the negative lens as the object for the positive lens:

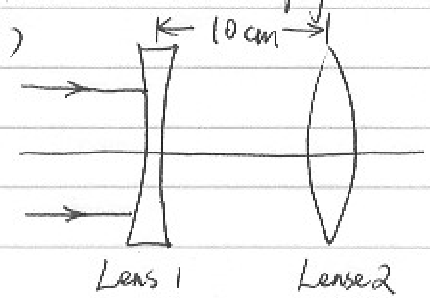
$$\frac{1}{s_2}+\frac{1}{s_2'}=\frac{1}{f_2} \Rightarrow \frac{1}{5}+\frac{1}{s_2'}=\frac{1}{20}, \quad \therefore s_2'=-6.67 \text{ cm}$$

Therefore, the equivalent focal length is -6.67 cm.

For a configuration of , the result is the same.

(b) There are two configurations depending on the order of lenses

(i)



$$\frac{1}{s_1}+\frac{1}{s_1'}=\frac{1}{f_1} \quad (s_1=\infty)$$

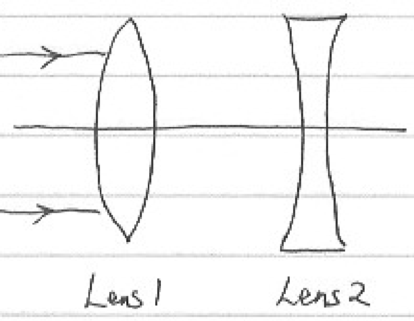
$$\therefore s_1'=f=-5 \text{ cm}$$

$$\frac{1}{s_2}+\frac{1}{s_2'}=\frac{1}{f_2}$$

$$\therefore \frac{1}{s_2'}=\frac{1}{20}-\frac{1}{15}=\frac{3-4}{60}=-\frac{1}{60}$$

$$\therefore s_2'=-60 \text{ cm} \quad \text{(equivalent focal length)}$$

(ii)



$$\frac{1}{s_1}+\frac{1}{s_1'}=\frac{1}{f_1} \quad (s_1=\infty)$$

$$\therefore s_1'=f_1=20 \text{ cm}$$

$$\frac{1}{s_2}+\frac{1}{s_2'}=\frac{1}{f_2}$$

$$\therefore \frac{1}{s_2'}=\frac{1}{-5\text{cm}}-\frac{1}{-10\text{cm}}=\frac{-1}{10}$$

$$\therefore s_2'=-10 \text{ cm.} \quad \text{(equivalent focal length)}$$

8. At first interface, use the refraction equation $\frac{n_1}{S_1} + \frac{n_2}{S_1'} = \frac{n_2 - n_1}{R}$

Here, $S_1 \cong 0$

$$\therefore \frac{n_2}{S_1'} = \frac{n_2 - n_1}{R} - \frac{n_1}{S_1} \cong -\frac{n_1}{S_1}$$

$$\therefore S_1' = -\frac{n_2}{n_1} S_1 \cong 0$$

$\therefore S_2 = \text{diameter of marble} - S_1' = (-2R) - 0 = -2R$

(R is negative for the 2nd surface)

At the second interface,

$$\frac{n_2}{S_2'} = \frac{n_2 - n_1}{R} - \frac{n_1}{-2R}$$

Where, $n_2 = 1.0$ (air), $n_1 = n = 1.6$ (refractive index of glass)

$$\therefore \frac{1}{S_2'} = \frac{1-n}{R} + \frac{n}{2R} = \frac{2-2n+n}{2R} = \frac{2-n}{2R}$$

$$\therefore S_2' = \frac{2R}{2-n} = \frac{2 \times (-1)\text{ cm}}{2-1.6} = \frac{-2}{0.4} = -5\text{ cm}$$

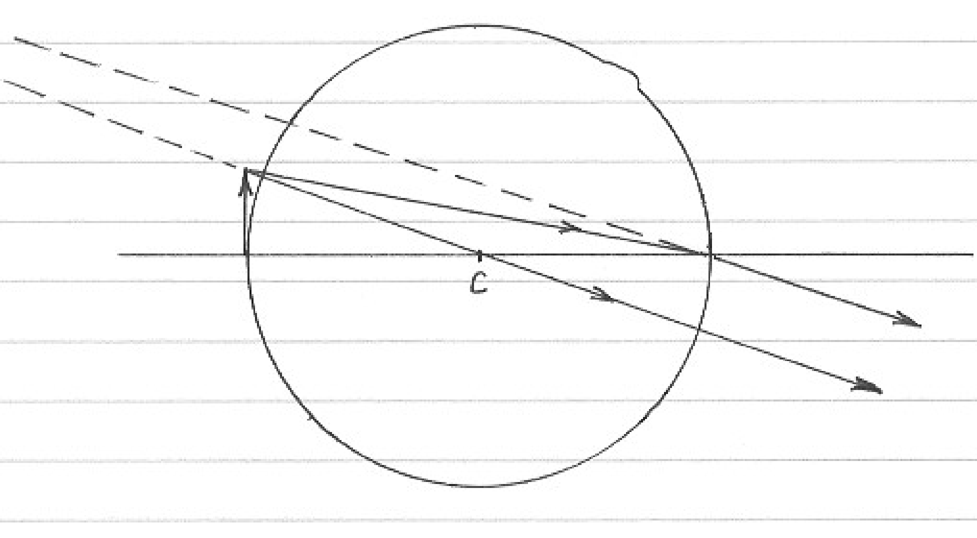
Magnification at first interface, $M_1 = +1$

Magnification at second interface $M_2 = -\frac{n_1 S_2'}{n_2 S_2} = -\frac{n S_2'}{S_2} = -\frac{n \cdot \frac{2R}{2-n}}{-2R}$

$$\therefore M = M_1 \cdot M_2 = \frac{n}{2-n}$$

M is independent of R for this geometry, $M = \frac{1.6}{2-1.6} = +4$

When $n \to 2$, $m \to \infty$, therefore, $n=2$ gives maximum magnification.



Note that $S_2' \to \infty$, when $n \to 2$.
So if $n=2$, rays emerge parallel.

If $n>2$, rays will cross to right.
→ real image and negative m.