Physics 3600 Optics and Photonics I
Winter 2006
Assignment # 3 Solutions

1. For $y = A\cos(2.01\times10^{7}x - 3.77\times10^{15}t)$ (x in meters and t in seconds)

   We know $y = A\cos(kx - \omega t)$

   (a) Propagation constant $k = 2.01\times10^{7}\ m^{-1}$

   (b) Angular frequency $\omega = 3.77\times10^{15}$ rad/sec

   (c) Wavelength in the medium
   $$\lambda = \frac{2\pi}{k} = \frac{2\pi}{2.01\times10^{7}} = 3.126\times10^{-7}\ m = 312.6\ nm$$

   (d) Frequency
   $$f = \frac{\omega}{2\pi} = \frac{3.77\times10^{15}}{2\pi} = 6\times10^{14}\ sec^{-1}$$

   (e) Period
   $$T = \frac{1}{f} = 1.67\times10^{-15}\ sec = 1.67\ fs$$

   (f) Velocity in the medium and propagation direction
   $$v = \lambda\cdot f = 3.126\times10^{-7}\cdot 6\times10^{14} = 1.8756\times10^{8}\ m/sec$$
   in $+\vec{x}$ - direction.

   (g) Refractive index of the medium
   $$n = \frac{c_0}{v} = \frac{3\times10^{8}}{1.8756\times10^{8}} = 1.6$$

   (h) Wavelength in free space
   $$\lambda_0 = n\lambda = 1.6\times 312.6\ nm = 500.16\ nm.$$

2. $E_e = 0.135\ W/cm^2$, $\lambda_{avg} = 700\ nm$

   (a) $$E_0 = \sqrt{\frac{2E_e}{\epsilon_0 c}} = \sqrt{\frac{2\times1350}{\epsilon_0 c}} = 1.01\times10^{3}\ V/m$$
   $$B_0 = \frac{E_0}{c} = 3.36\times10^{-6}\ T$$

   (b) $$n = \frac{E_e}{h\nu} = \frac{E_e}{hc/\lambda} = \frac{1350\ J/m^2\cdot s}{hc/(700\times10^{-9}m)} = 4.76\times10^{21}\ photons/m^2\cdot s$$

   (c) $$k = \frac{2\pi}{\lambda} = \frac{2\pi}{700\times10^{-9}} = 1.43\times10^{6}\ m^{-1} \text{ and } \nu = \frac{c}{\lambda} = 4.28\times10^{14}\ s^{-1}$$
   $$\therefore E = 1010\sin 2\pi(1.43\times10^{-6}\,r \pm 4.28\times10^{14}t)$$

3. $v_g = v_p - \lambda \dfrac{dv}{d\lambda}$ where $v_p = \dfrac{c}{n}$

(a) $\lambda = \dfrac{c}{\nu} = \dfrac{2\pi c}{\omega}$

$$\frac{dv}{d\lambda} = \frac{d(c/n)}{d(2\pi c/\omega)} = \frac{-c\,dn/n^2}{-2\pi c\,d\omega/\omega^2} = \frac{\omega^2}{2\pi n^2}\frac{dn}{d\omega}$$

Therefore, $v_g = v_p - \left(\dfrac{2\pi c}{\omega}\right)\left(\dfrac{\omega^2}{2\pi n^2}\right)\dfrac{dn}{d\omega} = v_p\left(1 - \dfrac{\omega}{n}\dfrac{dn}{d\omega}\right)$

(b) For normal dispersion, $\dfrac{dn}{d\lambda} < 0$, then $\dfrac{dn}{d\omega} > 0$

Therefore, $v_g < v_p$.

4. For Fresnel's biprism, Light from a small source S appears to come from two coherent, virtual sources, S_1 and S_2. (see Fig. 10-8 in the textbook, P&P, page 210)

The deviation angle $\delta_m \cong \alpha(n-1)$ for prisms of small prism angle α (read textbook, P&P, p. 116-118 for the derivation)

The virtual source separation a is: $a = 2d\,\delta_m = 2d\alpha(n-1)$

Bright fringe positions are

$$y_m = \frac{m\lambda(d+s)}{2d\alpha(n-1)}$$

where $m = 0, 1, 2, \cdots$

5. (a) $s = \dfrac{a\Delta y}{\lambda \Delta m} = \dfrac{0.05 \times 0.1}{600 \times 10^{-7} \times 1} = 83.3$ cm

(b) Path difference between beams is $\Delta = m\lambda$.

For beams with and without the plate, $\Delta_2 - \Delta_1 = (\Delta m)\lambda$

$\therefore \Delta m = \dfrac{\Delta_2 - \Delta_1}{\lambda} = \dfrac{nt - t}{\lambda} = \dfrac{t(n-1)}{\lambda} = \dfrac{100 \times 10^{-4} \times (1.5-1)}{600 \times 10^{-7}} = 83.3$ fringes

(c) $I = 4I_0 \cos^2\left(\dfrac{\pi a y}{\lambda s}\right) = 4I_0 \cos^2\left(\dfrac{\pi\Delta}{\lambda}\right)$, where $\Delta = \dfrac{ay}{s}$

At $\Delta = 0$, $I_{max} = 4I_0$

At peak half-maximum, $I = \dfrac{I_{max}}{2} = 2I_0$,

So, $2I_0 = 4I_0 \cos^2\left(\dfrac{\pi\Delta}{\lambda}\right)$, $\therefore \Delta = \dfrac{\lambda}{4} = 150$ nm

6. For normal incidence to achieve atireflecting, it requires

$$2 n_f t = (m+\tfrac{1}{2})\lambda$$

$$\therefore \quad t = \frac{(m+\frac{1}{2})\lambda}{2 n_f} \qquad (1)$$

For incidence at 45° to achieve atireflecting (minimal reflection), it requires:

$$2 n_f t \cos\theta_t = (m+\tfrac{1}{2})\lambda' \qquad (2)$$

By Snell's Law, $\sin 45° = n_f \sin\theta_t \Rightarrow \sin\theta_t = \frac{1}{\sqrt{2}\, n_f}$ (3)

Using the relation $\sin^2\theta_t + \cos^2\theta_t = 1$

We can obtain: $\cos^2\theta_t = 1 - \sin^2\theta_t = 1 - \frac{1}{2n_f^2} = \frac{2n_f^2 - 1}{2n_f^2}$

$$\therefore \quad \cos\theta_t = \frac{\sqrt{2n_f^2-1}}{\sqrt{2}\, n_f} \qquad (4)$$

Substituting (1) and (4) into (2):

$$\cancel{2 n_f} \cdot \frac{\cancel{(m+\frac{1}{2})}\lambda}{\cancel{2 n_f}} \cdot \frac{\sqrt{2n_f^2-1}}{\sqrt{2}\, n_f} = \cancel{(m+\tfrac{1}{2})}\lambda'$$

$$\therefore \quad \lambda' = \lambda \cdot \cos\theta_t$$

$$= \lambda \cdot \sqrt{\frac{2n_f^2-1}{2n_f^2}}$$

$$= 580 \cdot \sqrt{\frac{2\times 1.38^2 - 1}{2\times 1.38^2}}$$

$$= 498 \text{ nm}.$$