Physics 3600 Optics and Photonics I

Winter 2006

Assignment #4 Solutions

1. P&P, Problems 10-25, P. 223.

Consider the Newton's rings apparatus illustrated in Fig. 10-16 (P&P, P. 217). For the observation of reflected light, the path difference between reflected rays from the top and bottom surfaces Δ_r is $\lambda/2$ at point $t=0$.

Thus the condition for bright and dark fringes (P&P, p. 216, eq. 10-36):

$$2 n_f t + \Delta_r = \begin{cases} m\lambda & \text{bright} \\ (m+\frac{1}{2})\lambda & \text{dark} \end{cases} \qquad (1)$$

So for the bright fringes to occur, it must satisfy

$$2t_m = (m-\tfrac{1}{2})\lambda, \qquad m = 1, 2, 3, \cdots \qquad (2)$$

where, $n_f = 1$ for air; t_m is the air-film thickness corresponding to the mth bright ring. The center of the fringe pattern thus appears dark.

For light of wavelength 546-nm, $2t_m = (11-0.5)\lambda = 10.5\lambda_1$ (3)

For light of the other wavelength, $2t_m = (10-0.5)\lambda = 9.5\lambda_2$ (4)

From (3) and (4), we obtain: $10.5\lambda_1 = 9.5\lambda_2$

So, $\lambda_2 = \frac{10.5}{9.5}\lambda_1 = \frac{10.5}{9.5} \times 546\text{ nm} = 603.5\text{ nm}$

By using (3), we obtain $t_m = \frac{10.5}{2}\lambda_1 = \frac{10.5 \times 546\text{nm}}{2} = 2.87 \times 10^{-4}\text{ cm}$

From eq. (10-37) (P&P, p. 217): $R^2 = r_m^2 + (R - t_m)^2$ (5)

Since the air-film thickness t_m is small, the term t_m^2 is negligible in eq. (5) as compared with other terms in the equation.

So, $\cancel{R^2} = r_m^2 + \cancel{R^2} - 2Rt_m + \cancel{t_m^2}$

$$r_m^2 = 2Rt_m$$

Therefore, $r_m = \sqrt{2Rt_m} = \sqrt{2 \times (100) \times (2.87 \times 10^{-4})} = 0.240\text{ cm}.$

2. P&P, Problem 11-5, p. 244.

(a) When the gas is introduced into one arm of the Michelson interferometer, the resultant change in phase difference is

$$\Delta = 2nL - 2L = 2L(n-1) \qquad (1)$$

N fringes correspond to a change in phase difference

$$\Delta = N\lambda \qquad (2)$$

From (1) and (2), we obtain

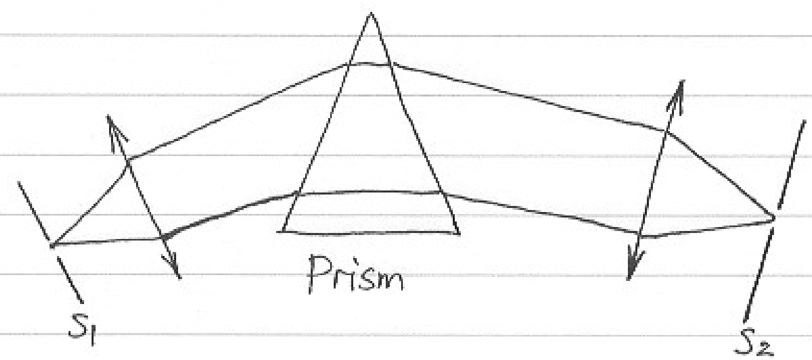
$$N\lambda = 2L(n-1) \quad \Rightarrow \quad n = 1 + \frac{N\lambda}{2L} \qquad (3)$$

(b) By using eq.(3), we obtain

$$1.00045 = 1 + \frac{N\cdot(589\times10^{-7}\,cm)}{2\times 10\,cm}$$

$$\therefore\ N = 153$$

3. P&P, Problem 12-10, p.264.



Wavelength dispersion than can pass through the "exit slit" is

$$\Delta\lambda = 20\,\mathring{A}/mm \times 0.2\,mm = 4\,\mathring{A}$$

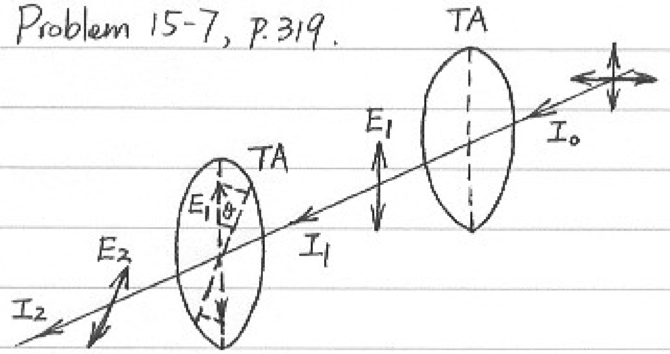
Thus, the coherence length l_t is (P&P, eq.(12-17), p.253)

$$l_t = \frac{\lambda^2}{\Delta\lambda} = \frac{(5\times10^{-5}\,cm)^2}{4\times10^{-8}\,cm} = 0.0625\,cm$$

The coherence time τ_0 is (P&P, eq.(12-16), p.253)

$$\tau_0 = \frac{l_t}{c} = \frac{0.0625\,cm}{c} = \frac{0.0625\,cm}{3\times10^{10}\,cm/s} = 2.08\times10^{-12}\,s$$

4. P&P, Problem 15-7, p.319.



(a) $I_1 = \frac{I_o}{2}$

$E_2 = E_1 \cos\theta$

$\therefore I_2 = (E_1 \cos\theta)^2 = I_1 \cos^2\theta = \frac{I_o}{2} \cos^2\theta$

(b) $\theta = 90° - 0° = 90°$

$\therefore I_2 = \frac{I_o}{2} \cos^2 90° = 0$

(c) With the addition of five polarizers, there are seven elements now.

$I_7 = I_6 \cos^2 15°$

$I_6 = I_5 \cos^2 15°$

$\vdots$

$I_1 = \frac{1}{2} I_o$

Therefore, $I_7 = (\cos^2 15°)^6 \cdot (\frac{1}{2} I_o) = 0.3298\, I_o \cong 33\%\, I_o$

5. P&P, Problem 16-1, p.346.

For Fraunhofer diffraction from a single slit, the condition for zeros of the irradiance is (P&P, eqs.(16-12) & (16-13), p.327)

$$m\lambda = b \sin\theta = b\frac{y}{f}$$

(a) For first minimum, $m=1$

So, $y = \frac{\lambda f}{b} = \frac{(546.1 \times 10^{-6} mm) \times 60 cm}{0.015 cm} = 2.18\ mm$

(b) $\Delta y = \frac{f\lambda \Delta m}{b}$ with $\Delta m = 2-1 = 1$

$\therefore \Delta y = 2.18\ mm.$