Physics 3600 Optics and Photonics I
Winter 2006
Assignment #5 Solutions

1. P&P, Problem 20-1, p. 423

$$r = 0 \Rightarrow n^2\cos\theta - \sqrt{n^2-\sin^2\theta} = 0$$

$$\therefore \quad n^2\cos\theta = \sqrt{n^2-\sin^2\theta}$$

$$n^4\cos^2\theta = n^2-\sin^2\theta$$

$$n^4\cos^2\theta - n^2 + \sin^2\theta = 0$$

$$\therefore \quad n^2 = \frac{-(-1)\pm\sqrt{(-1)^2-4\cos^2\theta\sin^2\theta}}{2\cos^2\theta} = \frac{1\pm\sqrt{1-4\cos^2\theta\sin^2\theta}}{2\cos^2\theta}$$

$$1-4\cos^2\theta\sin^2\theta = 1-4(1-\sin^2\theta)\sin^2\theta = 1-4\sin^2\theta+4\sin^4\theta = (2\sin^2\theta-1)^2$$

$$\therefore \quad n^2 = \frac{1\pm(2\sin^2\theta-1)}{2\cos^2\theta} = \begin{cases} \dfrac{2\sin^2\theta}{2\cos^2\theta} = \tan^2\theta & (\text{"+"}) \\[2ex] \dfrac{2\cos^2\theta}{2\cos^2\theta} = 1 & (\text{"−", trivial}) \end{cases}$$

2. P&P, Problem 20-11, p. 424

(a) External TM: $\theta_p = \tan^{-1} n = \tan^{-1} 2.42 = 67.55^\circ$

θ_c does not exist when $n_2 > n_1$

External TE: θ_p, θ_c : none

(b) Internal TM: $\theta_p' = \tan^{-1}(1/n) = \tan^{-1}(1/2.42) = 22.45^\circ$

$\theta_c = \sin^{-1}(1/n) = \sin^{-1}(1/2.42) = 24.41^\circ$

Internal TE: θ_p', none

$\theta_c = \sin^{-1}(1/n) = \sin^{-1}(1/2.42) = 24.41^\circ$

3. P&P, Problem 20-15, p.424

(a) For Fresnel rhomb, it requires $\phi_{TM} - \phi_{TE} = 90^\circ$ after two internal reflections or $\phi_{TM} - \phi_{TE} = 45^\circ$ after one internal reflection.

For $\theta > \theta_c = \sin^{-1}(1/1.65) = 37.3^\circ$

Using eqs. (20-34) and (20-35):

$$\phi_{TM} - \phi_{TE} = 2\tan^{-1}\left(\frac{\sqrt{\sin^2\theta - n^2}}{n^2 \cos\theta}\right) - 2\tan^{-1}\left(\frac{\sqrt{\sin^2\theta - n^2}}{\cos\theta}\right) = 45^\circ$$

where $n = \frac{n_2}{n_1} = \frac{1}{1.65}$ (internal reflections)

By trial and error, or aided by a calculator program or software, we find

$$\theta = 59^\circ 51'$$

(b) $5\% \cdot \theta = 0.05 \times 59.857^\circ = 2.993^\circ$

$\theta + 2.993^\circ = 62.85^\circ$ and $\Delta\phi = 41.16^\circ \times 2 = 82.32^\circ$

$\theta - 2.993^\circ = 56.864^\circ$ and $\Delta\phi = 48.50^\circ \times 2 = 97.00^\circ$

4. P&P, Problem 24-5, p.519

(a) $N.A. = \sqrt{n_1^2 - n_2^2} = \sqrt{1.53^2 - 1.39^2} = 0.6394$

(b) $n_0 \sin\theta_m = N.A. = 0.64$

$\therefore \theta_m = 39.75^\circ$

Acceptance angle (maximum cone angle) $= 2\theta_m = 79.5^\circ$

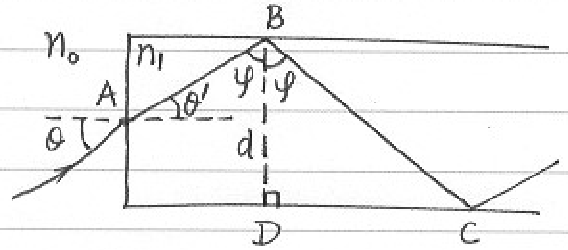
(c) $$\frac{1}{L_s} = \frac{1}{d\sqrt{\left(\frac{n_1}{n_0 \sin\theta}\right)^2 - 1}} = \frac{1}{(0.0025\,in)\sqrt{\left(\frac{1.53}{\sin 39.75^\circ}\right)^2 - 1}} = 184/in$$

or 6624 reflections in 3 ft.

At half θ_m, $\theta = 19.87^\circ$, then $\frac{1}{L_s} = 91.14\ in^{-1}$

or 3281 reflections in 3 ft.

5. P&P, Problem 24-6, p.519.



(a) From ΔBDC:

$$\cos\varphi = \frac{d}{X_s} \Rightarrow X_s = \frac{d}{\cos\varphi} \qquad (5.1)$$

Applying Snell's law at the entrance of the fibre:

$$n_0 \sin\theta = n_1 \sin\theta' = n_1 \sin(90° - \varphi) = n_1 \cos\varphi$$

As we know $n_0 = 1$ (air),

$$\therefore \sin\theta = n_1 \cos\varphi \Rightarrow \cos\varphi = \frac{\sin\theta}{n_1} \qquad (5.2)$$

So Eq. (5.1) becomes: $X_s = \dfrac{d}{\frac{\sin\theta}{n_1}} = \dfrac{n_1 d}{\sin\theta}$

(b) $X_t = X_s \cdot X(\text{number of skips})$

$$= X_s \cdot \left(\frac{L}{L_s}\right) = \frac{n_1 d}{\sin\theta}\left[\frac{L}{d\sqrt{\left(\frac{n_1}{n_0 \sin\theta}\right)^2 - 1}}\right] = \frac{n_1 L}{\sqrt{\left(\frac{n_1}{n_0}\right)^2 - \sin^2\theta}} \overset{n_0=1}{=\!=} \frac{n_1 L}{\sqrt{n_1^2 - \sin^2\theta}}$$

(c) $$X_s = \frac{n_1 d}{\sin\theta} = \frac{1.5 \times 50\,\mu m}{\sin 10°} = 432\ \mu m$$

$$L_s = d\sqrt{\left(\frac{n_1}{\sin\theta}\right)^2 - 1} = 50\,\mu m \cdot \sqrt{\left(\frac{1.5}{\sin 10°}\right)^2 - 1} = 429\ \mu m$$

$$X_t = \frac{n_1 L}{\sqrt{n_1^2 - \sin^2\theta}} = \frac{1.5 \times 10 m}{\sqrt{1.5^2 - \sin^2 10°}} = 10.07\ m$$