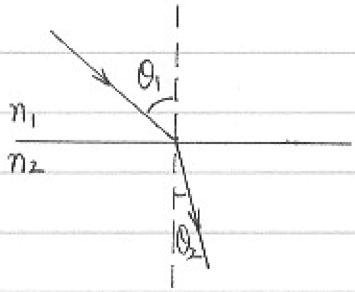
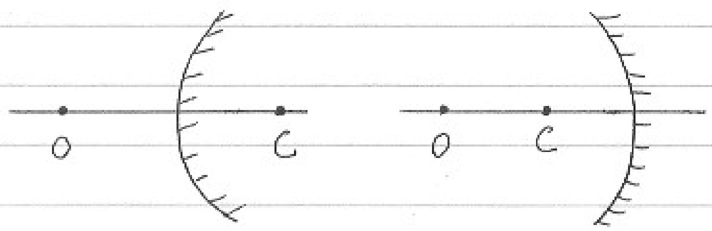
Tutorial : 1. Review on Geometrical Optics



Snell's Law: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

Total internal reflection: when ray travelling from medium 1 to medium 2 with $n_1 > n_2$, the critical angle of incidence $\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$



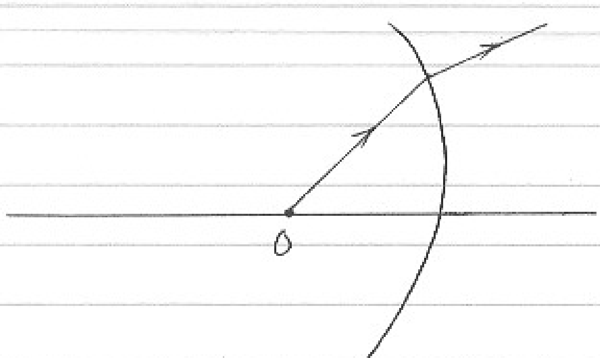
Reflection at a spherical surface

Mirror equation: $\frac{1}{s} + \frac{1}{s'} = -\frac{2}{R}$

or $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$

magnification $m = -\frac{s_i}{s_o}$

Sign convention: ① light travels from left → right
② s: +, object is to the left of V
③ s': + image is to the left of V
④ R: + when C is to the right of V



Refraction at a spherical surface

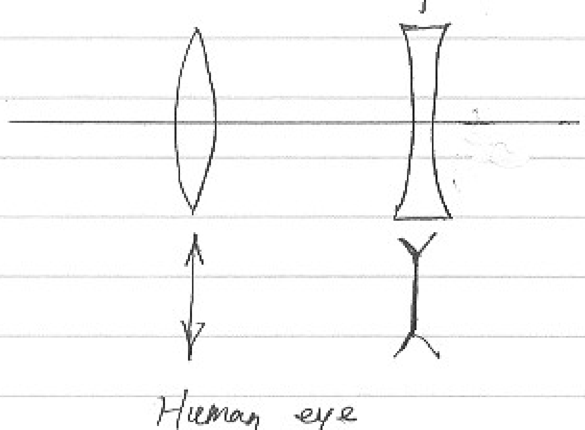
Refraction equation: $\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R}$

lateral magnification: $m = -\frac{n_1 s'}{n_2 s}$

Sign convention: s: + for real objects & images
− for virtual objects & images

positive lens $f > 0$

negative lens $f < 0$



Lensmaker's equation

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$\frac{1}{f} = \frac{n_2 - n_1}{n_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

magnification: $m = -\frac{s'}{s}$

$\left.\begin{array}{l}\text{Human eye}\\ \text{Simple magnifiers}\\ \text{Microscopes}\\ \text{Telescopes.}\end{array}\right\}$ class notes.

2. Interference

1-D wave equation : $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$ (v: speed)

3-D wave equation : $\nabla^2 \psi = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2}$

$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ Laplacian operator

Harmonic waves :

$$\begin{cases} y = A \begin{smallmatrix}\sin\\ \cos\end{smallmatrix} [k(x \pm vt)] \\ y = A \begin{smallmatrix}\sin\\ \cos\end{smallmatrix} [2\pi(\frac{x}{\lambda} \pm \frac{t}{T})] \\ y = A \begin{smallmatrix}\sin\\ \cos\end{smallmatrix} [(kx \pm \omega t)] \end{cases} \Longleftarrow \begin{cases} K = \frac{2\pi}{\lambda} \\ \nu = \frac{\omega}{2\pi} = \frac{1}{T} \\ v = \lambda \nu = \frac{\omega}{k} \end{cases}$$

In complex form, $\psi = A e^{i(\vec{k}\cdot\vec{r} - \omega t)}$

For EM waves, ψ stands for either the varying electric or magnetic fields. The plane EM wave can be written as :

$$\vec{E} = \vec{E}_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$
$$\vec{B} = \vec{B}_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

Superposition principle ($\psi = \psi_1 + \psi_2$)

$\hookrightarrow$ Superposition of EM waves $\begin{cases} \vec{E} = \vec{E}_1 + \vec{E}_2 \\ \vec{B} = \vec{B}_1 + \vec{B}_2 \end{cases}$

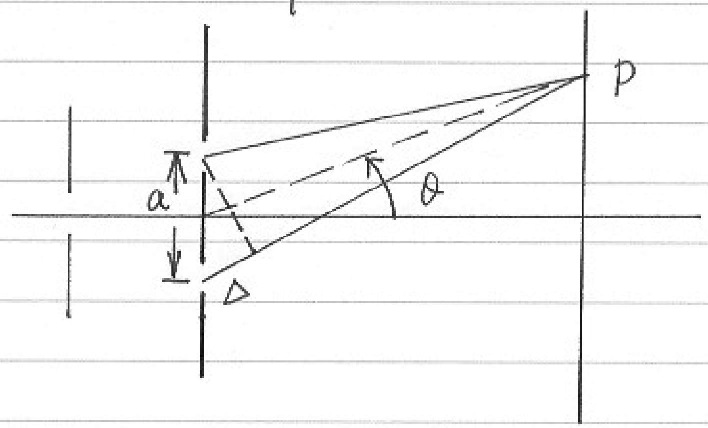
Phase and group velocities

- Two beam interference

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta \Longrightarrow I = 4 I_0 \cos^2\left(\frac{\delta}{2}\right)$$

Interfering beams of equal amplitude

- Young's double slit experiment



Optical path difference $\Delta \cong a\sin\theta = \begin{cases} m\lambda & \text{(Constructive interference)} \\ (m+\frac{1}{2})\lambda & \text{(Destructive interference)} \end{cases}$

Phase difference $\delta = k\cdot\Delta = \left(\frac{2\pi}{\lambda}\right)\cdot a\sin\theta$

$$I = 4I_0\cos^2\left(\frac{\pi\Delta}{\lambda}\right) = 4I_0\cos^2\left(\frac{\pi a\sin\theta}{\lambda}\right)$$

Double-slit interference with virtual source:

Lloyd's mirror;

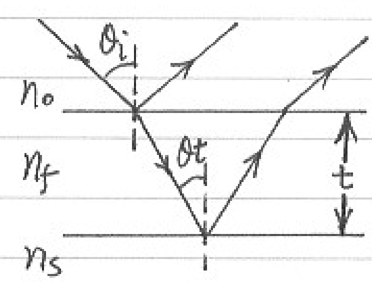
Fresnel's mirror;

Fresnel's biprism.

- Interference in dielectric films / fringes of equal thickness

$$\Delta_p + \Delta_r = \begin{cases} m\lambda & \text{(constructive interference)} \\ (m+\frac{1}{2})\lambda & \text{(destructive interference)} \end{cases}$$

Optical path difference $\Delta_p = 2n_f\, t\cos\theta_t$



Δ_r: equivalent path difference arising from phase change on reflection.

If $n_f > n_o$ and $n_f > n_s$, $\Delta_r = \frac{\lambda}{2}$

If $n_o < n_f < n_s$ (two external reflections), $n_o > n_f > n_s$ (two internal reflections): $\Delta_r = 0$

Examples:

Air wedge
Newton's rings
Film - thickness measurement
Michelson interferometer

- Multiple - beam interference and Fabry - Perot interferometer
- Coherence
- Polarized Light

3. Diffraction

Diffraction from many slits

Therefore the irradiance is

$$I = I_0 \underbrace{\left(\frac{\sin\beta}{\beta}\right)^2}_{\text{diffraction envelope}} \underbrace{\left(\frac{\sin N\alpha}{\sin\alpha}\right)^2}_{\text{interference}} \qquad \text{where} \quad \begin{aligned}\beta &= \tfrac{1}{2}kb\sin\theta \\ \alpha &= \tfrac{1}{2}ka\sin\theta\end{aligned}$$

Discussion :

- When $N=1$, $I = I_0\left(\frac{\sin\beta}{\beta}\right)^2\left(\frac{\sin N\alpha}{\sin\alpha}\right)^2 = I_0\left(\frac{\sin\beta}{\beta}\right)^2$, single slit diffraction
- When $N=2$, $I = I_0\left(\frac{\sin\beta}{\beta}\right)^2\left(\frac{\sin N\alpha}{\sin\alpha}\right)^2 = I_0\left(\frac{\sin\beta}{\beta}\right)^2\left(\frac{\sin 2\alpha}{\sin\alpha}\right)^2$

  $$= I_0\left(\frac{\sin\beta}{\beta}\right)^2\left(\frac{2\sin\alpha\cos\alpha}{\sin\alpha}\right)^2 = 4I_0\left(\frac{\sin\beta}{\beta}\right)^2\cos^2\alpha$$

  double slit diffraction
- Consider the interference factor $\left(\frac{\sin N\alpha}{\sin\alpha}\right)^2$:

  $$\lim_{\alpha\to m\pi}\frac{\sin N\alpha}{\sin\alpha} = \lim_{\alpha\to m\pi}\frac{N\cos N\alpha}{\cos\alpha} = \pm N \qquad (m=0,\pm1,\pm2,\cdots)$$

  $\Rightarrow$ Intensity of principal maxima of interference pattern $\propto N^2$

With $\alpha = \frac{p\pi}{N}$ or $a\sin\theta = \frac{p\lambda}{N}$ $\quad p=0,\pm1,\pm2,\cdots\cdots$

$\left(\alpha = \frac{1}{2}ka\sin\theta = p\frac{\pi}{N} \Rightarrow \cancel{\tfrac{1}{2}}\cdot\frac{\cancel{2}\cancel{\pi}}{\lambda}a\sin\theta = \frac{p\cancel{\pi}}{N} \Rightarrow a\sin\theta = \frac{p\lambda}{N}\right)$

Principal maxima occur for $p = 0, \pm N, \pm 2N, \cdots$

minima occur for p = all other values

($N-1$ minima exist between principal maxima)

$N-2$ small, secondary maxima exist between principal maxima

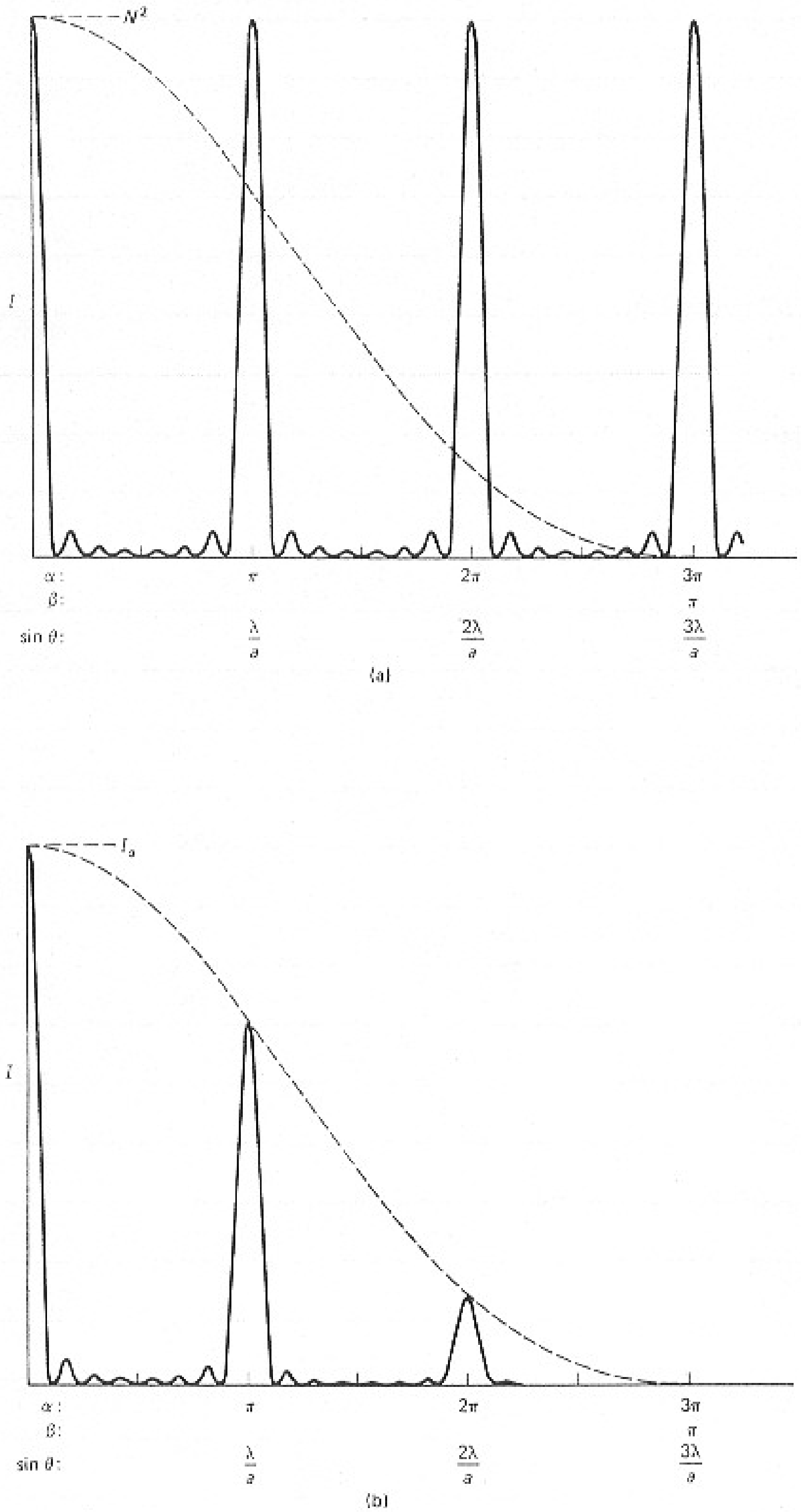


Figure 16-14 (a) Interference (solid line) and diffraction (dashed line) functions plotted for multiple-slit Fraunhofer diffraction when $N = 8$ and $a = 3b$. (b) Irradiance function for the multiple slit of (a). The irradiance is limited by the diffraction envelope (dashed line).

4. Maxwell's theory and polarization

- EM waves:

$$c^2 = \frac{1}{\varepsilon_0 \mu_0} \quad \text{for vacuum} \qquad \varepsilon \equiv \text{permittivity}$$

$$v^2 = \frac{1}{\varepsilon \mu} \quad \text{in general} \qquad \mu \equiv \text{permeability}$$

$$n = \frac{c}{v}$$

- Fresnel equations

  TE (transverse electric) and TM (transverse magnetic) modes

- External and internal reflection

  Brewster condition

- Conservation of energy

- Evanescent waves

- Complex refractive index and reflection from metals.

5. Fibre-optical light guides

   - Basics

     Light coupling into optical fibres

     N.A.

     Ls (skip distance, axial distance)

     Allowed modes

   - Attenuation — extrinsic losses & intrinsic losses

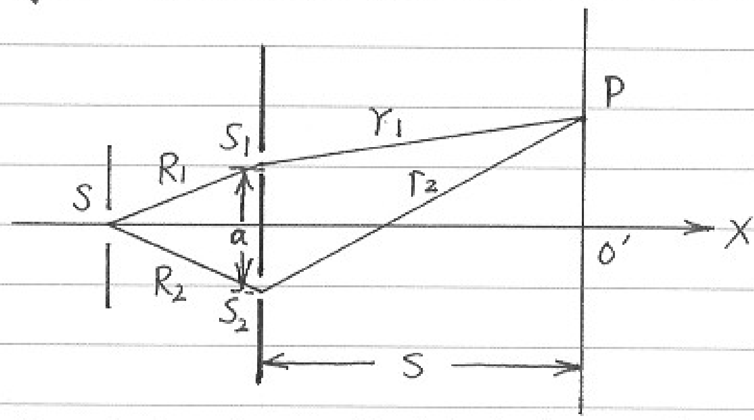
   - Distortion

     — modal distortion, material dispersion, waveguide dispersion

     — Pulse distortion limits transmission frequency and information rate.

$$\delta\tau > \frac{T}{2} \quad \text{or} \quad \delta\tau > \frac{1}{2\nu} \qquad (\nu: \text{frequency})$$

Practice 1. Young's double slit experiment



Discussion of the changes of the interference fringes in Young's double slit experiment under the following situations:
(a) light source S moves upward; (b) light source moves towards double slits; (c) Screen moves away from the double slits;
(d) if the slit separation is doubled; (e) A transparent film with $n>1$ is placed at S_1.

Solution:

(a) If light source S moves upward, phase difference at the double slits is not zero, i.e., $\frac{2\pi}{\lambda}(R_1-R_2)$, where $R_1<R_2$. If we trace the movement of the original zero-order fringe at O', only points at $r_1>r_2$ can achieve zero-order fringe. So the fringes will move down.

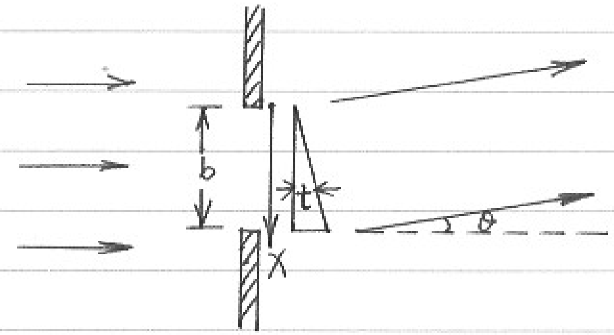
(b) If the light source S moves towards the double slits, the location and the separation of the fringes will not be changed.

(c) If screen moves away from the double slits, $\Delta y=\frac{\lambda S}{a}$, fringe separation will increase, however, the location for central zero-order fringe will not move. (for monochromatic light)

(d) If the slit separation is doubled, the fringe separation will be half of the original value.

(e) If a transparent film with $n>1$ is placed at S_1, the optical path S_1P is increased. Considering point P of the original j th order, to keep the optical path difference $S_1P-S_2P=$ constant, r_2 should increase, so P will move upward.

Practice 2. Diffraction from a single slit



A plane wave of propagation constant K reaches a slit of width b. The slit is covered by a transparent wedge with thickness proportional to the distance from its apex, i.e., $t = \gamma x$ (γ is constant). The refractive index of the wedge is n. Show the expression of irradiance along θ direction $I \propto \frac{\sin^2(\beta b)}{(\beta b)^2}$.
Give the expression of β in terms of K, n, γ and θ.

Solution:

If there is no wedge, the irradiance for the diffraction from a single slit is:

$$I = I_0 \frac{\sin^2\left(\frac{1}{2}Kb\sin\theta\right)}{\left(\frac{1}{2}Kb\sin\theta\right)^2}$$

Since the prism angle γ is small, light will deviate $\delta = (n-1)\gamma$ toward the prism base after propagating through the prism.
The result is that all rays will deviate the same angle toward the prism base while the irradiance has the same distribution as for the diffraction from single slit. We can just replace θ with $\theta - \delta$.

$$I = I_0 \frac{\sin^2\left\{\frac{1}{2}Kb\sin[\theta-(n-1)\gamma]\right\}}{\left\{\frac{1}{2}Kb\sin[\theta-(n-1)\gamma]\right\}^2}$$

$$\therefore \beta = \frac{1}{2}K\sin[\theta-(n-1)\gamma]$$