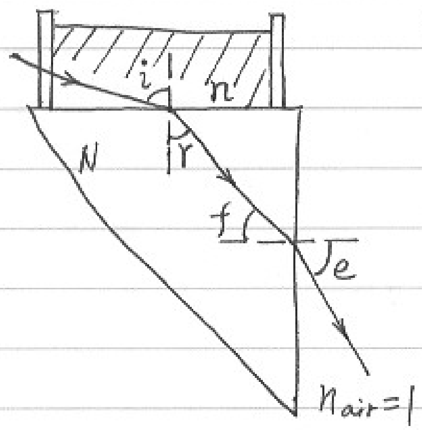
Physics 3600 Optics and Photonics I
Winter 2006
Term Test One Solutions

1. See solutions to Problem #4 of Assignment #1 or Problem #2 of Assignment #2

2.



2a) Snell's Law at the boundary of Liquid and prism gives:

$$N \sin r = n \sin i$$

$$\therefore \frac{n}{N} = \frac{\sin r}{\sin i} \qquad (1)$$

From drawing $i > r$, and $i, r < 90^\circ$,

$$\therefore \sin i > \sin r \qquad (2)$$

From inequality of (2) $\Rightarrow \frac{\sin r}{\sin i} < 1 \qquad (3)$

From eqs. (1) and (3) $\Rightarrow \frac{n}{N} < 1$ or $n < N$

2b) Snell's Law at the boundary of prism and air:

$$N \sin\left(\frac{\pi}{2} - r\right) = \sin e \qquad (4)$$

Note: $f = \frac{\pi}{2} - r$, $n_{air} = 1$

$$\therefore N\sqrt{1 - \sin^2 r} = \sin e \qquad (5)$$

From Eq. (1): $\sin^2 r = \left(\frac{n}{N}\right)^2 \sin^2 i \qquad (6)$

From Eqs. (5) and (6): $N\sqrt{1 - \left(\frac{n}{N}\right)^2 \sin^2 i} = \sin e$

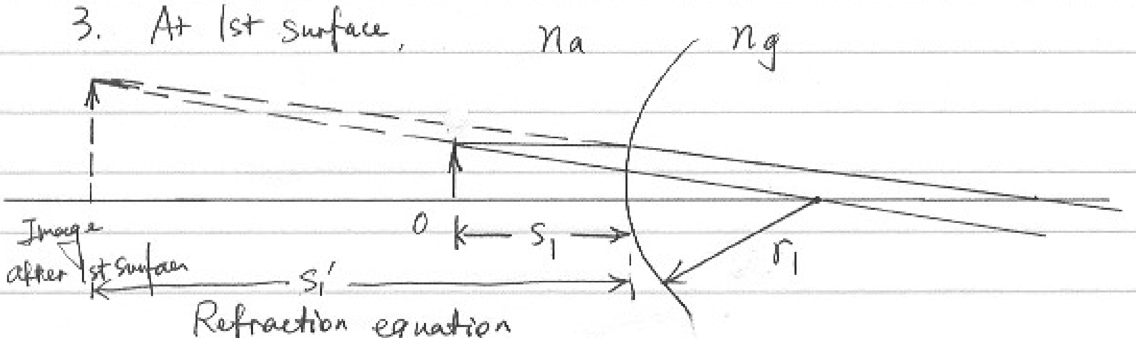
$$N^2\left[1 - \left(\frac{n}{N}\right)^2 \sin^2 i\right] = \sin^2 e$$

$$n^2 \sin^2 i = N^2 - \sin^2 e$$

$$\therefore n = \sqrt{\frac{N^2 - \sin^2 e}{\sin^2 i}}$$

For $i \sim 90^\circ$, $\sin i \approx 1$, therefore $n = \sqrt{N^2 - \sin^2 e}$

3. At 1st surface,



Refraction equation

$$\frac{n_a}{s_1} + \frac{n_g}{s_1'} = \frac{n_g - n_a}{r_1}$$

Where $n_a = 1$, $n_g = 1.5$, $r_1 = 10$ cm, $s_1 = 8$ cm.

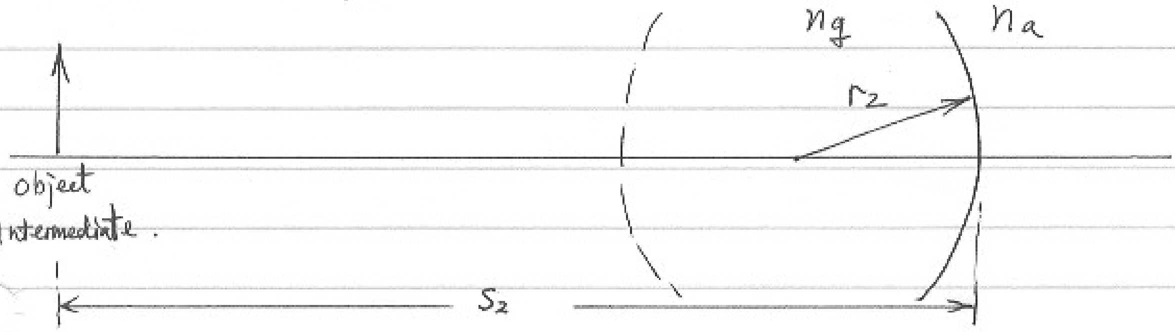
$$\therefore \frac{1}{8} + \frac{1.5}{s_1'} = \frac{1.5 - 1.0}{10}$$

$$\therefore s_1' = -20 \text{ cm}$$

magnification $m_1 = -\dfrac{n_a s_1'}{n_g s_1} = -\dfrac{-20}{1.5 \times 8} = \dfrac{5}{3}$

Virtual image, upright.

At 2nd surface



$$\frac{n_g}{s_2} + \frac{n_a}{s_2'} = \frac{n_a - n_g}{r_2}$$

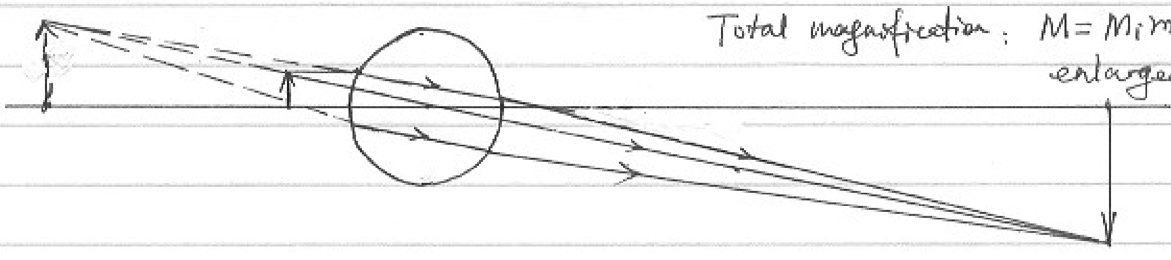
where $s_2 = -(s_1' - 2|r_1|) = -(-20\text{cm} - 2 \times 10\text{cm}) = +40$ cm

$r_2 = -10$ cm

$$\therefore \frac{1.5}{40} + \frac{1}{s_2'} = \frac{1 - 1.5}{-10}$$

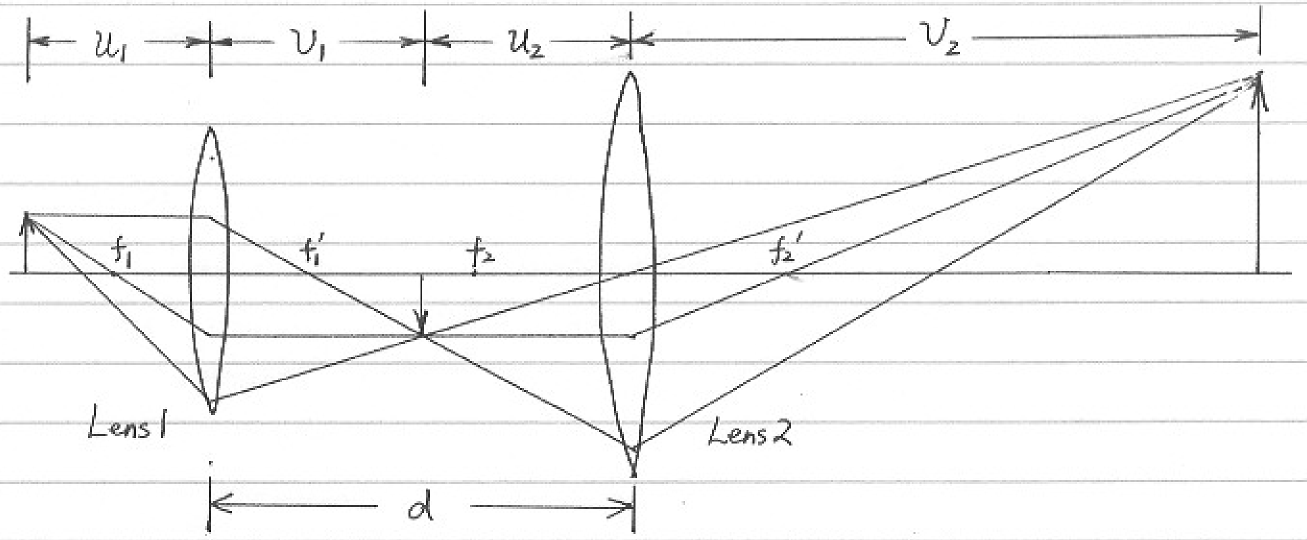
$$\therefore s_2' = +80 \text{ cm.}$$

magnification $m_2 = -\dfrac{n_g s_2'}{n_a s_2} = -\dfrac{1.5 \times 80}{40} = -3$



Total magnification: $M = m_1 m_2 = \frac{5}{3} \cdot (-3) = -5$.
enlarged, inverted image.

4.(a)



In order to design the system of two lenses with magnification of +30, we need:

1) The object distance to lens 1 being larger than the focal length of that lens ($f_1 < u_1$);
2) The image produced by the lens 1 should be located between the two lenses' focal lengths;
3) The separation of the two lenses must be larger than the sum of their focal lengths ($f_1 + f_2 < d$).

Given the distance of object from the first lens: $u_1 = 1$ mm.
One can choose $f_1 = 0.5$ mm $< u_1$.

Using equation $$\frac{1}{u_1} + \frac{1}{v_1} = \frac{1}{f} \qquad (1)$$

$$\Rightarrow \quad v_1 = \frac{u_1 f_1}{u_1 - f_1} = \frac{1\,\text{mm} \times 0.5\,\text{mm}}{1\,\text{mm} - 0.5\,\text{mm}} = 1\,\text{mm}$$

The total magnification is given by

$$M_{Tot} = m_1 \cdot m_2 \qquad (2)$$

Where m_1 and m_2 are the lateral magnifications of the lens 1 and 2, respectively.

$$m_1 = -\frac{v_1}{u_1} = -1$$

$$m_2 = -\frac{v_2}{u_2}$$

Given $M_{TOT} = +30$, We can obtain $m_2 = -30$ or $v_2 = 30u_2$

Using lensmaker's equation $\frac{1}{u_2} + \frac{1}{v_2} = \frac{1}{f_2}$

$$\frac{1}{u_2} + \frac{1}{30u_2} = \frac{1}{f_2}$$

$$\therefore f_2 = \frac{30}{31} u_2$$

We need to calculate f_2. while we know $f_1 + f_2 < d$ and $f_1 = 0.5$ mm.

One can choose $d = 11$ mm, So, $u_2 = d - u_1 = 10$ mm

$$\therefore f_2 = \frac{30}{31} u_2 = \frac{30}{31} \cdot 10 \text{ mm} = 9.67 \text{ mm}.$$

The image is formed at $v_2 = 30u_2 = 300$ mm (on right side of lens 2)
Therefore, the distance between object and image is

$$u_1 + d + v_2 = 1 \text{ mm} + 11 \text{ mm} + 300 \text{ mm} = 312 \text{ mm}.$$

(b) Using a medium like oil between the sample and objective lens (lens 1 in (a)) helps to have more light refract from the sample to the objective lens because ($n_{oil} > n_{glass} > n_{air}$).
(Read textbook P&P, Section 6-5 on Numerical Aperture, P.136, Fig.6-25)