PHYSICS 3600 TERM TEST MARCH 24, 2006

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ATTEMPT ALL QUESTIONS

Name: Solutions **Student #:** ____________

CONSTANTS AND FORMULAE

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$c = 2.998 \times 10^8 \, m/s$$

$$I = 4I_0 \cos^2\left(\frac{\pi\Delta}{\lambda}\right) = 4I_0 \cos^2\left(\frac{\pi a y}{\lambda s}\right)$$

$$I = I_o\left(\frac{\sin^2\beta}{\beta^2}\right), \quad \beta = \frac{1}{2}kb\sin\theta$$

$$I = 4I_o\left(\frac{\sin^2\beta}{\beta^2}\right)\cos^2\alpha, \quad \beta = \frac{1}{2}kb\sin\theta, \quad \alpha = \frac{1}{2}ka\sin\theta$$

$$I = I_o\left(\frac{\sin^2\beta}{\beta^2}\right)\left(\frac{\sin^2 N\alpha}{\sin^2\alpha}\right), \quad \beta = \frac{1}{2}kb\sin\theta, \quad \alpha = \frac{1}{2}ka\sin\theta$$

$$e^{i\alpha} = \cos\alpha + i\sin\alpha$$
$$\sin^2\theta + \cos^2\theta = 1$$
$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$
$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

Name:__________________________ **Student #:**____________________

1. (25 points) In an interference experiment of the Young type, the distance between slits is 0.5 mm, and the wavelength of the light is 600 nm.

   1a) If it is desired to have a fringe spacing of 1 mm at the screen, what is the proper screen distance?

   1b) If a thin plate of glass (n = 1.50) of thickness 100 microns is placed over one of the slits, what is the lateral fringe displacement at the screen?

   1c) What path difference corresponds to a shift in the fringe pattern from a peak maximum to the (same) peak half-maximum?

(1a) $S = \frac{a\,\Delta y}{\lambda\,\Delta m} = \frac{0.05 \times 0.1}{600 \times 10^{-7} \times 1} = 83.3 \text{ cm}$

(1b) Path difference between beams is $\Delta = m\lambda$

For beams with and without the plate, $\Delta_2 - \Delta_1 = (\Delta m)\lambda$

$\therefore \Delta m = \frac{\Delta_2 - \Delta_1}{\lambda} = \frac{nt - t}{\lambda} = \frac{t(n-1)}{\lambda} = \frac{100 \times 10^{-4} \times (1.5-1)}{600 \times 10^{-7}} = 83.3$ fringes

(1c) $I = 4I_0 \cos^2\left(\frac{\pi a y}{\lambda s}\right) = 4I_0 \cos^2\left(\frac{\pi \Delta}{\lambda}\right)$, where $\Delta = \frac{ay}{s}$

At $\Delta = 0$, $I_{max} = 4I_0$

At peak half-maximum, $I = \frac{I_{max}}{2} = 2I_0$

So, $2I_0 = 4I_0 \cos^2\left(\frac{\pi\Delta}{\lambda}\right)$

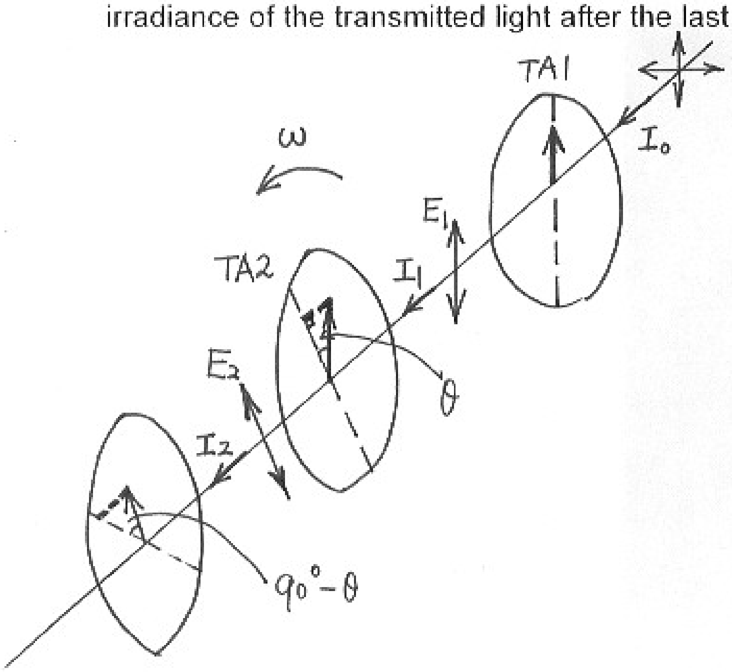
$\therefore \Delta = \frac{\lambda}{4} = 150 \text{ nm}$

Name:__________________________ **Student #:**____________________

2. (25 points) A dichroic polarizer, which is rotating around the propagation direction of light with an angular frequency of ω, locates between two dichroic polarizers of orthogonal transmission axes. All dichroic polarizers can be assumed perfect, that is, each passes 50% of the incident unpolarized light. Using a sketch, show that the transmitted light has been modulated to have a frequency of four times of the angular frequency of the rotation, i.e.,

$$I = \frac{I_0}{16}[1 - \cos(4\omega t)]$$

where I_0 is the irradiance of the incident light before the first polarizer and I is the irradiance of the transmitted light after the last polarizer.



$$I_1 = \frac{I_0}{2}$$

$$E_2 = E_1 \cos\theta$$

$$\therefore I_2 = (E_1 \cos\theta)^2 = I_1 \cos^2\theta = \frac{I_0}{2}\cos^2\theta$$

$$E_3 = E_2 \cdot \cos(90^\circ - \theta) = E_2 \sin\theta$$

$$\therefore I_3 = (E_2 \sin\theta)^2$$

$$= I_2 \sin^2\theta$$

$$= \frac{I_0}{2}\cos^2\theta \sin^2\theta$$

$$= \frac{I_0}{2} \cdot \frac{1+\cos 2\theta}{2} \cdot \frac{1-\cos 2\theta}{2}$$

$$= \frac{I_0}{8}(1 - \cos^2 2\theta)$$

$$= \frac{I_0}{8}\left(1 - \frac{1+\cos 4\theta}{2}\right)$$

$$= \frac{I_0}{8} \cdot \frac{1-\cos 4\theta}{2}$$

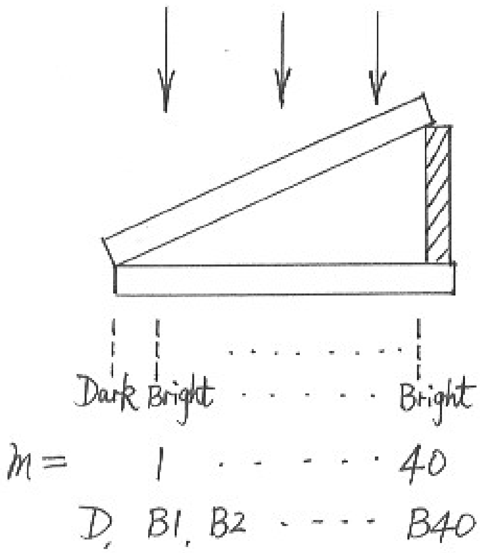
$$= \frac{I_0}{16}(1 - \cos 4\theta)$$

As we know $\theta = \omega t$

$$\therefore I = I_3 = \frac{I_0}{16}[1 - \cos(4\omega t)]$$

Name:____________________________ **Student #:**______________________

3. (20 points) Two microscope slides are placed together but held apart at one end by a thin piece of tin foil. Under sodium light (589 nm) normally incident on the air gap formed between the slides, one observes exactly 40 bright fringes from the edges in contact to the edge of the tin foil. Determine the thickness of the foil.



Condition for bright fringes:

$$2n_f t + \Delta_r = m\lambda$$

Where $\Delta_r = \frac{\lambda}{2}$

For 40 bright fringes correspond to $m = 1, 2, 3, \cdots 40$,

At 40 th bright fringe,

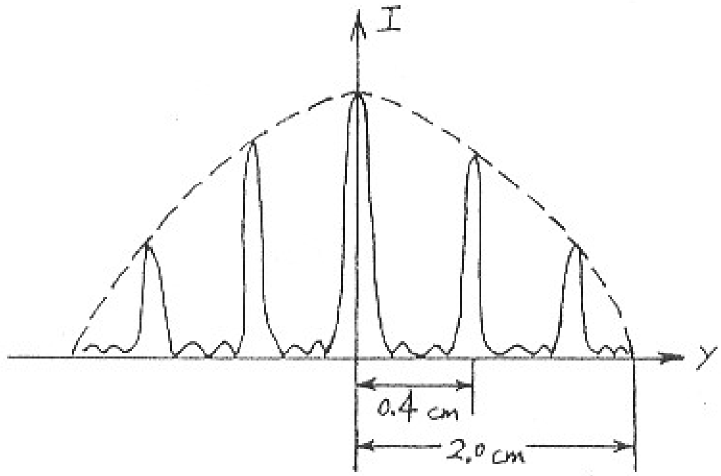
$$2t = \left(m - \frac{1}{2}\right)\lambda$$
$$= (40 - 0.5)\lambda$$
$$= 39.5\lambda$$
$$\therefore \quad t = \frac{39.5}{2}\lambda$$
$$= \frac{39.5}{2} \times 589 \times 10^{-7}$$
$$= 1.16 \times 10^{-3} \text{ cm}$$

Name:__________________________ **Student #:**____________________

4. The following figure shows the variation of irradiance with vertical distance from the axis at the screen, which has been measured from N identical parallel slits. The distance from the slits to the screen is 20 meters. The wavelength of the light to pass through the slits is 600 nm. Suppose each slit has a slit width *b* with slit separation *a*.

4a) (18 points) Find out the values of N, *b*, and *a*.

4b) (12 points) Write the expression for the envelope (i.e. the dash line) in the figure and explain its physical implication.



(4a) There are $N-2$ small, secondary maxima between the neighbouring principal maxima, From the figure, we know $N=4$.

The first zero irradiance of the dash line depends on the factor of diffraction envelope, i.e., when $\beta = \pi$, $I=0$. From the figure, we know $y = 2.0$ cm, so $\sin\theta = \frac{2\text{ cm}}{20\text{ m}} = 1.0\times10^{-3}$

$$\therefore\ \beta = \frac{1}{2}kb\sin\theta = \frac{1}{2}\frac{2\pi}{\lambda}b\sin\theta = \frac{\pi}{\lambda}b\sin\theta = \pi$$

$$\therefore\ b = \frac{\lambda}{\sin\theta} = \frac{6\times10^{-7}}{10^{-3}} = 6\times10^{-4}\text{ m} = 0.6\text{ mm}$$

When the principle maxima occurs, $\lim_{\alpha\to m\pi}\frac{\sin N\alpha}{\sin\alpha} = \pm N$, $m=0,\pm1,\pm2\cdots$

First principal maxima corresponds to $\alpha=\pi$ ($m=1$), $\alpha = \frac{\pi}{\lambda}a\sin\theta = \pi$

where $\sin\theta = \frac{0.4\text{ cm}}{20\text{ m}} = 0.2\times10^{-3}$

$$\therefore\ a = \frac{\lambda}{\sin\theta} = \frac{6\times10^{-7}\text{ m}}{0.2\times10^{-3}} = 3\times10^{-3}\text{ m} = 3\text{ mm}$$

(4b) The dash line is the diffraction envelope of the irradiance, i.e.

$$I \propto \left(\frac{\sin\beta}{\beta}\right)^2 = \left[\frac{\sin\left(\frac{\pi b}{\lambda}\sin\theta\right)}{\frac{\pi b}{\lambda}\sin\theta}\right]^2, \quad \text{where } \sin\theta \cong 10^{-3}$$

The diffraction from many slits is the modulation from single-slit diffraction on multiple beam interference