

Physics 4300. Assignment 3.

1 Assume that the atmospheric Ekman layer over the earth's surface at latitude 45°N can be modeled with an eddy viscosity $\nu_E = 10 \text{ m}^2/\text{s}$. If the geostrophic velocity above the layer is 10 m/s and uniform, what is the vertically integrated flow across the isobars (pressure contours)? Is there any vertical velocity?

2 You are working for a company that plans to deposit high-level radioactive wastes on the bottom of the ocean, at a depth of 3000 m . This site (latitude: 33°N) is known to be at the center of a permanent counterclockwise vortex. Locally, the vortex flow can be assimilated to a solid-body rotation with angular speed equal to 10^{-5} s^{-1} . Assuming a homogeneous ocean and a steady, geostrophic flow, estimate the upwelling rate at the vortex center. How many years will it take for the radioactive wastes to arrive at the surface? Take $f = 8 \times 10^{-5} \text{ s}^{-1}$ and $\nu = 10^{-2} \text{ m}^2/\text{s}$.

3 Between 15°N and 45°N , the winds over the North Pacific Ocean consist mostly of the easterly trades (15°N to 30°N) and the mid-latitude westerlies (30°N to 45°N). An adequate representation is

$$\tau^x = \tau_0 \sin\left(\frac{\pi y}{2L}\right), \quad \tau^y = 0 \quad \text{for} \quad -L \leq y \leq L,$$

with $\tau_0 = 0.15 \text{ N/m}^2$ (maximum wind stress) and $L = 1670 \text{ km}$. Taking $\rho_0 = 1028 \text{ kg/m}^3$ and the value of the Coriolis parameter corresponding to 30°N , calculate the Ekman pumping. Which way is it directed? Calculate the vertical volume flux over the entire 15° – 45°N strip of the North Pacific (width = 8700 km). Express your answer in sverdrup units ($1 \text{ sverdrup} = 1 \text{ Sv} = 10^6 \text{ m}^3/\text{s}$).

4 In a certain region, at a certain time, the atmospheric temperature along the ground decreases northward at the rate of 1°C every 35 km , and there are good reasons to assume that this gradient does not change much with height. If there is no wind at ground level, what are the wind speed and direction at an altitude of 2 km ? To answer, take latitude = 40°N , mean temperature = 290 K , and uniform pressure on the ground.

Hint: use the thermal expansion coefficient $\alpha = 1/T$ (ideal gas) to relate temperature gradient to density gradient.

5

A cruise to the Gulf Stream at 38°N provided a cross-section of the current, which was then approximated to a two-layer model (Figure 15-18) with a warm layer of density $\rho_1 = 1025 \text{ kg/m}^3$ and depth $h(y) = H - \Delta H \tanh(y/L)$, overlying a colder layer of density $\rho_2 = 1029 \text{ kg/m}^3$. Taking $H = 500 \text{ m}$, $\Delta H = 300 \text{ m}$, and $L = 60 \text{ km}$ and assuming that there is no flow in the lower layer and that the upper layer is in geostrophic balance, determine the flow pattern at the surface. What is the maximum velocity of the Gulf Stream? Where does it occur? Also, compare the jet width (L) to the radius of deformation.

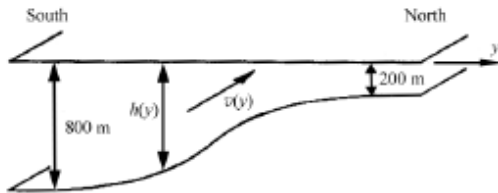


Figure 15-18 Schematic cross-section of the Gulf Stream, represented as a two-layer geostrophic current (Analytical Problem 15-2).