EXPERIMENTS ON INSTABILITY OF COLUMNAR VORTEX PAIRS IN ROTATING FLUID

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We present results from a new series of experiments on the geophysically important issue of the instability of anticyclonic columnar vortices in a rotating fluid in circumstances such that the Rossby number exceeds unity. The vortex pair consisting of a cyclonic and an anticyclonic vortex is induced by a rotating flap in a fluid which is itself initially in a state of solid-body rotation. The anticyclonic vortex is then subject to either centrifugal or elliptical instability, depending on whether its initial ellipticity is small or large, while the cyclone always remains stable. The experimental results demonstrate that the perturbations due to centrifugal instability have a typical form of toroidal vortices of alternating sign (rib vortices). The perturbations due to elliptical instability are of the form of sinuous deformation of the vortex filament in the plane of maximal stretching which corresponds to the plane of symmetry for the vortex pair. The initial perturbations in both cases are characterized by a definite wave number in the vertical direction. The characteristics of the unstable anticyclone are determined by the main nondimensional parameter of the flow – the Rossby number. The appearance of both centrifugal and elliptical instabilities are in accord with the predictions of theoretical criteria for these cases.

1. INTRODUCTION

Quasi-two-dimensional vortices are well known coherent structures in geophysical turbulence. These structures are abundant in both the atmosphere and the oceans and they play an important role in the global dynamics of these geophysical fluids. Large scale vortices are influenced by the Earth’s rotation. The vortices of smaller scale may also be embedded in rotating and/or strained flow fields due to the presence of other vortices. These factors may be crucial in determining the stability characteristics of such coherent structures. Columnar vortices may, in particular, be subject to the centrifugal (inertial) instability due to the background rotation (see e.g., Smyth and Peltier, 1994; Carnevale et al., 1997; Afanasyev and Peltier, 1998; Potylitsin and Peltier, 1998, 1999, 2002) or elliptical instability due to the background strain (see e.g., Moore and Saffman, 1975; Waleffe, 1990; Eloy and Le Dizes, 1999, 2001). Both centrifugal and elliptical instabilities are essentially three-dimensional. The perturbations

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of the vortex due to the centrifugal instability usually take the form of axisymmetric
toroidal vortices which have a mushroom-like shaped cross section (Fig. 1a). The
presence of background strain induces at first order an elliptic deformation of
the vortex core. This deformation leads to the development of elliptical instability
in the form of sinuous deformation of the vortex filament in the plane of maximal
straining (Fig. 1b).

Apart from these three-dimensional instabilities strictly two-dimensional instability
also exists. This instability is often called barotropic instability. Due to its strictly
two-dimensional character the barotropic instability is not affected by background
rotation. However, the effect of background rotation can create the conditions for
the development of the barotropic instability via the modification of the primary
vortex by centrifugal instability. In general, barotropic instability may occur when
vorticity changes sign at some distance from the center of the vortex. The vortices
in which the vorticity is single signed and decreases with distance from the center
are proved to be stable according to the Rayleigh inflection point theorem as well as
nonlinear stability theorems (see e.g., Drazin and Reid, 1981; Dritschel, 1988;
Carnevale and Shepherd, 1990). Isolated vortices (with zero net circulation) cannot
have single signed vorticity profile and are subject to barotropic instability (as well
as to other forms of instability in the presence of rotation and/or strain).

The combined effect of barotropic and centrifugal instability was considered in
laboratory experiments by Afanasyev and Peltier (1998) and in numerical simulations
by Orlandi and Carnevale (1999). These studies were motivated by the earlier rotating
tank experiments of Kloosterziel and van Heijst (1991). In their experiments vortices
were produced using the so-called stirring technique. The fluid inside a hollow cylinder
was stirred and the cylinder was then withdrawn from the fluid. The resulting vortex

FIGURE 1  Sketch of typical centrifugal (a) and elliptical (b) instabilities. (a) The primary centrifugal insta-
bility (2) of an initially columnar anticyclonic vortex (1) is of the form of toroidal vortices with mushroom-
like cross section. The Kelvin-Helmholtz-type (shear) secondary instability (3) occurs due to the modification of
the azimuthal velocity field by the radial motions (conserving angular momentum) induced by primary centri-
fugal instability. (b) The strain field in the horizontal plane is shown for the case of elliptical instability.
consisted of a patch of vorticity of one sign surrounded by a ring of opposite vorticity. The vortices with anticyclonic rotation split into two dipoles which subsequently moved apart ("quadrupolar instability") while the vortices with cyclonic rotation transformed into a structure with a stable core, so-called tripole (the tripole is in fact a variety of a quadrupole, see e.g., Voropayev and Afanasyev (1994) for general classification of vortex multipoles). The difference in the behavior of cyclonic and anticyclonic vortices was hypothesized to be the effect of centrifugal instability which was present only for the anticyclones.

An interesting geophysical example which demonstrates the difference between cyclones and anticyclones was provided recently by Potylitsin and Peltier (1998) for an atmospheric flow. Von Karman vortex streets are often observed to develop in the lee of the mountainous Jan Mayen island in the Norwegian Sea. It is clearly visible in satellite images of the flow provided by Potylitsin and Peltier (1998) (see also Etling, 1990) that it is often the case that anticyclonic vortices do not appear in the vortex streets. This is explained by the effect of destruction of anticyclonic vortices by the centrifugal instability similar to that observed in the rotating tank experiments. An example of the effect of the background rotation on the wake behind a solid body was recently provided in the large-scale laboratory experiment on the "Coriolis platform" by Spedding and Fincham (1999). In their experiment, the asymmetry of the wake was clearly seen in the vorticity maps of the flow when the ratio of the Coriolis parameter to buoyancy frequency was sufficiently large, provided that the Rossby number was greater than unity. The latter experiment indicates that the stratification which is often important in the geophysical flows effectively suppresses the instability. This also confirms the previous theoretical analysis by Potylitsin and Peltier (1998).

The experiments by Afanasyev and Peltier (1998) present an attempt to demonstrate centrifugal instability in controlled laboratory conditions and to clarify the mechanism by which the essentially three-dimensional centrifugal instability causes the two-dimensional barotropic instability which, in turn, destroys the vortex core. In these experiments the vortex in the rotating tank was induced by the hollow cylinder rotating in anticyclonic sense. The flow in the annulus around the cylinder immediately became subject to centrifugal instability while at the periphery of the vortex two-dimensional barotropic instability developed. This secondary barotropic instability developed due to the modification of the initial vorticity profile by the primary centrifugal instability. A sketch clarifying the topology of this flow is given in Fig. 1a. If the cylinder was withdrawn vertically from the tank, it was this secondary barotropic instability that destroyed the core of the vortex. The essential drawback of these experiments was that the core of the vortex was modelled by the solid cylinder such that the deformations of the core were not allowed. The main aim of the present work therefore is to extend these results in demonstrating the destabilizing effect of rotation to "free" vortices as well as vortex structures, namely vortex pairs (dipoles).

Stability of vortex pairs in nonrotating environment was studied both experimentally (Thomas and Auerbach, 1994; Leweke and Williamson, 1998) and numerically (Billant et al., 1999). Different modes of instability were found whose characteristics were consistent with those predicted by the elliptical instability theory. This instability is caused by the mutual staining of vortices in the pair. It is clearly interesting to examine experimentally the stability of the vortex pair in the rotating fluid where in addition to the elliptical instability the centrifugal instability may be important.
In the context of the experimental investigation of the elliptical instability it is worth mentioning a series of experiments by different authors on a vortex in an elliptical container (Malkus, 1989; Gledzer and Ponomarev, 1992; Manasseh, 1996; Eloy et al., 2000) and also direct numerical simulations by Mason and Kerswell (1999) for the same geometry of the flow. In the latter experiments the flow was created inside the rotating deformable cylinder of elliptic cross section. Different modes of elliptical instability were observed. The essential limitation of these experiments was, of course, the presence of a solid boundary around the vortex so that the large amplitude development of perturbations was not allowed. In this context the study of elliptical instability in a free flow, as attempted herein, seems advantageous. It is also of interest to generate an elliptically deformed vortex in a rotating environment in order to observe the effect of background rotation on the development of elliptical instability.

An important control parameter in determining the stability of a columnar vortex in a rotating environment is clearly the Rossby number \( Ro = \omega / 2 \Omega \), which represents the ratio of vorticity \( \omega \) in the column to the background vorticity \( 2 \Omega \). The behavior of flows for limiting cases in which \( Ro \) is either very small or very large, is relatively clear. When \( Ro \ll 1 \) the background rotation tends to two-dimensionalize (and thus stabilize) the vortical flow, an effect that is well understood on the basis of the Taylor-Proudman theorem (see e.g., Carnevale et al., 1997). In the opposite limit the flow doesn’t “feel” the rotation at all and there is therefore no essential difference between cyclones and anticyclones. Our intention herein is to focus upon the interesting intermediate regime, namely that in which \( Ro \geq 1 \), in which case the flows are far from the geostrophic regime (which is characterized by the balance of Coriolis and pressure forces for \( Ro \ll 1 \)). The recent theoretical article by Sipp and Jacquin (2000) gives a criterion for the flow to be unstable with respect to centrifugal perturbations on the basis of a local stability analysis (geometrical optics method) which allows one to consider the evolution of perturbations on individual streamlines (e.g., Lifschitz and Hameiri, 1991). In particular this criterion is a generalization of the criterion obtained earlier by Kloosterziel and van Heijst (1991). For circular flow the criterion simply states that the flow undergoes centrifugal instability if there exists a streamline where the absolute vorticity \( \omega + 2 \Omega \) changes sign. In terms of the Rossby number this means that the anticyclonic vortex is unstable for \( Ro \geq 1 \) (we assume further on that the background vorticity \( \Omega \) is negative so that the Rossby number for anticyclonic vortices is positive). A criterion for elliptical instability was derived by Leblanc and Cambon (1997) on the basis of local stability analysis. The criterion states that the instability occurs for the values of Rossby number in the interval \( Ro = 2 \pm D / 2 \Omega \) where \( D \) is the background strain rate. The maximum growth rate of instability occurs at \( Ro = 2 \). This result was obtained for wavevector of perturbations directed strictly along the rotation axis. This result was generalized recently by Le Dizes (2000) for arbitrary directions of wavevector on the basis of local stability analysis. It was shown that the instability domain extends towards larger values of the Rossby number when nonzero angles of the wavevector are allowed. It was also argued by Waleffe (1990), Eloy et al. (2000) and Le Dizes (2000) that the modes of the elliptical instability are, in fact, combinations of Kelvin (inertial) waves which resonate with the intrinsic mode corresponding to the elliptical deformation of the core of the vortex by background strain field. This approach appeared to be very fruitful in the interpretation of the experimental results by Eloy et al. (2000) on the unstable flow in elliptical cavity.
In a new sequence of experiments reported herein we chose a method similar to that used by Thomas and Auerbach (1994) to generate the barotropic vortices in a rotating tank. A flap was rotated in water a certain angle (about 45°) about the vertical axis in cyclonic or anticyclonic sense. When the flap starts its rotation the starting vortex is created at the lee side of the flap. When the flap stops the stopping vortex is induced at the opposite side of the flap. These vortices form an asymmetrical vortex pair which then propels itself forward from the flap. The interesting feature of these vortices is that the starting and stopping vortices differ from each other. While their intensity is not very different (the stopping vortex being slightly weaker), their ellipticity differs significantly. The starting vortex is almost circular and of negligible ellipticity initially. The stopping vortex, on the other hand, is created in the strain field of the already existing starting vortex. As a result the stopping vortex is elliptical to a significant degree. The anticyclonic vortex in the pair is then subject to instability while the cyclonic vortex remains stable (at least when the background rotation is not too weak). When the flap rotates in a cyclonic (anticyclonic) sense the resulting anticyclonic vortex is the starting (stopping) vortex and is therefore circular (elliptic). In spite of the obvious limitations of this experimental setup which did not allow us to vary the ellipticity of vortices, we believe that these experiments do provide considerable insight into the dynamics of both centrifugal and elliptical instability of free barotropic vortices in a rotating fluid.

2. LABORATORY APPARATUS AND TECHNIQUE

Our experiments were carried out in a rectangular Perspex tank of dimensions $80 \times 80 \times 15$ cm mounted on a rotating turntable (Fig. 2). The tank was rotated about a vertical axis through its center in an anticlockwise direction with the rotation rate $\Omega$ between $0.03 \text{s}^{-1}$ and $1.85 \text{s}^{-1}$. The tank itself was filled with a homogeneous fluid with a working depth of 14.5 cm and rotated until a nearly solid-body rotation was established. The flows in the tank were then generated by a thin plate (flap) placed vertically near the center of the tank and rotated in a clockwise or counterclockwise direction (Fig. 3). The frame supporting an electric motor with the flap on its axis was installed on the rotating table so that initially the flap was at rest with respect to the rotating tank. The dimensions of the flap and its rotation speed were fixed in our experiments. The space between the lower edge of the flap and the bottom of the tank was approximately 0.1 cm.

The working fluid consisted of a water solution of the pH indicator thymol-blue which was used for the visualization of the flows (e.g., Baker, 1966; Voropayev and Afanasyev, 1994). This solution is of an orange-yellow color in its neutral state. For flow visualization a DC voltage was applied, so that the flap constituted a negative electrode whereas a copper plate at the side of the tank constituted a positive electrode. Due to the electrochemical reaction, the solution near the flap becomes basic and, as a result, changes color to blue. Thus a layer of dark blue fluid is formed around the flap, making the flow pattern visible and providing a good contrast for photography. A camera mounted above the tank or at the side of the tank in the rotating reference frame was employed to record the patterns of dyed fluid that were formed by the flow. After each experiment it took only a few minutes for the diffusive
FIGURE 2  Sketch of the experimental set-up. A thin copper flap (1) is fixed to the axis of the motor (2). The Plexiglas tank (3) stands on the rotating table (4). The flow is recorded by CCD or photo cameras (5) in the rotating frame. The copper plate (6) is used as a positive electrode while the flap represents a negative electrode for the flow visualization in the solution of the pH indicator.

FIGURE 3  Generation of asymmetric vortices by a rotating flap. If the flap rotates in the same sense as the rotating platform the circular starting vortex is anticyclonic (A) while the elliptic stopping vortex is cyclonic (C).
chemical reaction with the acidic ambient fluid (an acid was added to the fluid in the tank) to restore the working fluid to its original yellow color.

The horizontal velocity field in the flow was measured using a PIV technique. A description of the method and general technique is given by Fincham and Spedding (1997) and Pawlak and Armi (1998). The seeding particles were polyamid spheres of mean diameter 50\,\mu m.

3. EXPERIMENTAL RESULTS AND INTERPRETATION

The rotating flap induces an asymmetric vortex pair consisting initially of an almost circular starting vortex and a deformed stopping vortex of lower intensity. By changing the sense of rotation of the flap either the starting or the stopping vortex can be made anticyclonic. The initial characteristics of the vortices in the pair were measured by PIV method. The results of vorticity measurements showed that the stopping vortex was about 20\% weaker than the starting vortex. Since the rotation rate and geometry of the flap were fixed, and therefore the values of the relative vorticity of vortices in the rotating frame were the same in all of our experiments, the initial value of the Rossby number for vortices was varied by varying the rotation rate of the tank between the experiments. We believe that the process of generation of vortices are not affected by the background rotation since the flow is initially two-dimensional (apart from narrow Ekman layer at the bottom) and the initial characteristics of vortices are therefore independent on the background rotation rate. This was confirmed by the results of measurements which show that the initial Rossby number changes linearly with $\Omega^{-1}$.

The Rossby number was estimated by the peak value of vorticity in the core of each vortex when the pair was formed in some short period of time after the flap stopped. The Rossby number is then given by the expression of the form

$$Ro = \frac{c}{2\Omega}$$

where the mean values of the coefficient $c$ were measured for starting vortex, $c = c_1 = 3.4 \pm 0.4 \, s^{-1}$, and for stopping vortex, $c = c_2 = 2.7 \pm 0.3 \, s^{-1}$. The anticyclonic vortex in the pair is then subject to either centrifugal or elliptical instability.

3.1 Anticyclonic Starting Vortex: Centrifugal Instability

In the first series of experiments that we performed the starting vortex was made anticyclonic so that the initial ellipticity of this vortex was negligible. The conditions were therefore more favorable for the development of the centrifugal instability. A typical evolution of the unstable flow is illustrated by the sequence of digitized video frames shown in Fig. 4. The perturbations develop in the anticyclonic vortex while its cyclonic neighbor remains stable and preserves its columnar form. The perturbations initially have the form of axisymmetric rib vortices of alternating sign. These vortices have typical mushroom-like shape in cross section (Fig. 4b).

During the finite amplitude stage of the instability development the flow in the horizontal plane is characterized by strong jet-like perturbations with dipolar vorticity
The example of velocity and vorticity field in the horizontal transection of the flow obtained by the PIV method is shown in Fig. 5. The jets with peak values of vorticity 2–4 times greater than the mean vorticity in the core of the vortex are clearly seen in Figs. 5a and b. These jets develop at the periphery of the vortex and are not symmetric with respect to the center of the vortex. Since the primary centrifugal instability is axisymmetric and does not produce strong vertical component of vorticity, it can hardly be seen in the velocity and vorticity fields that are shown in Fig. 5. The jet-like perturbations can therefore be explained by the development of a secondary Kelvin-Helmholtz type instability. This instability forms due to the modification of the radial distribution of vertical vorticity in the vortex by the primary centrifugal instability which involves radial motions. This effect was demonstrated earlier in the experiments with rotating cylinder (see Figs. 6 and 7 in Afanasyev and Peltier, 1998) and in the numerical simulations (Orlandi and Carnevale, 1999). This secondary instability was shown to destroy the core of the isolated vortex (when the cylinder was withdrawn from the tank).

In the present experiments the destruction of the vortex has not been observed. This is most likely due to the stabilizing action of the cyclonic vortex in the pair. The perturbations of the anticyclonic vortex eventually decay when the Rossby number based on the evolving peak vorticity in the vortex falls to a level below the threshold for centrifugal instability, $Ro < 1$ (Fig. 6). The restabilized anticyclone has a new mean Rossby number of approximately 0.4–0.8. This result agrees well with the results of numerical simulations by Carnevale et al. (1997). It is clearly seen in Fig. 6 that while the value of the nondimensional vorticity in the anticyclone is subject to strong variations during the development of the instability, the vorticity in the cyclonic neighbor remains much less disturbed.

The vertical wavelength of the primary instability was measured in our experiments as a function of the initial Rossby number $Ro$ (Fig. 7). The centrifugal instability with distinct wavelength was observed to develop in the range of the Rossby number between 1 and 3.5 for the anticyclonic vortex. The wavelength of the instability was almost constant over the entire range of values of the Rossby number. At higher values of the Rossby number, $Ro > 3.5$, the toroidal rib vortices which are typical for centrifugal instability were not observed. Instead, spiral type perturbations rather developed in the vortex core (Fig. 8). These perturbations then effectively destroyed the vortex.
FIGURE 5  Velocity (arrows) and vorticity (color) fields measured in the horizontal plane for the case of centrifugal instability. Experimental parameters: \( R_0 = 1.55 \); \( \Omega = 1.10 \text{s}^{-1} \); (a) \( t = 7 \text{s} \), (b) \( t = 11 \text{s} \), (c) \( t = 23 \text{s} \). Vorticity is given in nondimensional form \( \omega/2\Omega \). The large arrow at the top of each frame represents the velocity scale, 2 cm/s. Distance is in pixels, 1 pixel = 0.042 cm.
FIGURE 6 The variation of the absolute values of nondimensional peak vorticity in the cyclonic vortex (stars) and anticyclonic vortex (circles) with time. Time is nondimensionalized by the period of rotation of the tank. Solid lines represent the fit to the experimental data by polynomials of the second order.

FIGURE 7 Nondimensional wavelength of the initial perturbations for different values of the Rossby number for centrifugal (circles) and elliptical (stars) instability. Solid line represents a fit to the experimental data for elliptical instability.
3.2 Anticyclonic Stopping Vortex: Elliptical Instability

In the second series of experiments the flap was rotated in the anticyclonic sense. The starting vortex was therefore cyclonic while the deformed stopping vortex was anticyclonic. In this case the conditions were more favorable for the development of the elliptical instability due to the fact that anticyclone was subject to strain field. A typical evolution of the flow for two values of the Rossby number is shown in Figs. 9 and 10. The core of the anticyclone becomes unstable. It bends in the sinusoidal manner in the plane parallel to the plane of symmetry of the pair (perpendicular to the direction of view in Figs. 9 and 10) which corresponds to the direction of maximum straining. Note that this initial perturbation of the vortex tube is similar to that observed by Malkus (1989) in a closed elliptical container. It is therefore clear that the instability observed on our experiments can be characterized as an elliptical instability. The deformed vortex tube forms dipoles in cross section (see Figs. 10b and c). The tube then wraps around the stable cyclonic vortex. The development of the elliptical instability can also be observed in the sequence of video frames in Fig. 11 which shows the top view of the flow. The initial deformation of the core of the anticyclone can be clearly seen in Fig. 11b. The tip of the deformed vortex tube which is shown by arrows in Figs. 11c–e eventually wraps around the cyclone. The initial stage of the evolution of the anticyclone can be compared qualitatively with the results of numerical simulations of Carnevale et al. (1997) for their case of large Rossby number and strong initial perturbation of the vortex where a similar form of the development of the perturbation was observed.
The instability which can be characterized as a typical elliptical instability developed very effectively for higher values of the Rossby number $Ro > 2$, while for smaller values of the Rossby number both the vertical wavelength and the amplitude of the perturbations were relatively small (comparable to the size of the core of the vortex) and vertical motions developed in the core of the anticyclone (Fig. 12). These motions resulted in the convergence of dye in the middle of the vortex. The wavelength of elliptical instability was measured as a function of the initial value of the Rossby number (Fig. 7). In contrast to the wavelength of the centrifugal instability which stays almost constant, the wavelength of the elliptical instability increases when the Rossby number increases. This dependence can be described by the linear function of the form $\lambda/H = 0.12 Ro$.

The example of vorticity and strain fields for the flow subject to elliptical instability is shown in Fig. 13. The vorticity field shown in Figs. 13a and b demonstrates that the anticyclonic vortex is significantly strained. The black contour in Figs. 13c and d indi-
The areas of the flow where the criterion for the elliptical instability, $Ro = 2 \pm D/2\Omega$ (Leblanc and Cambon, 1997) is satisfied. These areas are located directly in the core of the anticyclonic vortex.

Typical velocity and vorticity fields in the vertical plane are shown in Fig. 14. Although the measurements in the vertical plane are not highly reliable in terms of accuracy because the velocity component perpendicular to the illuminated plane was...
FIGURE 14  Velocity (arrows) and vorticity (color) fields measured in the vertical plane perpendicular to the direction of motion of the dipole for the case of elliptical instability. Experimental parameters: $Ro=2.7; \Omega = 0.50 \text{s}^{-1};$ (a) $t=11 \text{s},$ (b) $t=16 \text{s}.$ Distance is in pixels, 1 pixel $= 0.038 \text{cm}.$
very strong, they can be used to demonstrate the vertical picture of disturbances in the dipole. The dipole moves towards the observer and passes through the illuminated vertical plane where the velocity field is measured by the PIV method. Two plates in Fig. 14 correspond to the consecutive sections of the frontal part of the dipole. Uniform flow to the right which corresponds to the wrapping of the anticyclone around the cyclonic vortex, is observed in all of the frames. It is clearly seen that the vorticity forms distinct structures which are inclined at the angle of approximately 45° to the horizontal.

4. DISCUSSION

The experimental observations described herein provide a picture of the development of the two main types of instability in the unbounded anticyclonic vortex. When the initial ellipticity of the vortex is small, the centrifugal instability is dominant. Intense perturbations spontaneously appear and amplify in the flow. These motions are initially of the form of axisymmetric toroidal vortices of alternating sign. It is clear from Fig. 4 that the perturbations due to the centrifugal instability are located around the edge of the vortex. This is consistent with the results of numerical simulations by Smyth and Peltier (1994) and Potylitsin and Peltier (1998) where the perturbation kinetic energy was shown to be concentrated at the edge, hence the name “edge” mode for the centrifugal instability.

The numerical results by Potylitsin and Peltier (1998) give in particular the maximum destabilization of the vortex for the value of nondimensional vertical wavenumber equal to approximately 1.6. This result was obtained for a rotating Kelvin-Helmholtz vortex train. The wavenumber was nondimensionalized by the diameter of the vortex. We may attempt to determine the initial diameter of the vortex by the dimensions of the dyed core. Dyed fluid comes from the boundary layer at the plate where the vorticity is generated, therefore indicates (at least initially) the vortex tube. The estimate of a tube diameter of 0.5 cm gives the nondimensional wavelength $\lambda/H \approx 0.07$ for the wavenumber of 1.6. This value is in good agreement with the values of the wavelength for the centrifugal instability measured in our experiments (see Fig. 7).

The appearance of the jet-like perturbations at the periphery of the unstable vortex (Fig. 5) is an indication of the secondary Kelvin-Helmholtz type instability. This type of instability appears due to the modification of the vorticity distribution in the primary vortex by the motions due to the centrifugal instability (see e.g., Afanasyev and Peltier, 1998; Orlandi and Carnevale, 1999).

It is interesting to note that the vortex pair keeps its translational motion during the development of the perturbations in the anticyclone. The pair describes in fact a circular path of large radius (Fig. 5c) due to slightly different intensity of cyclone and anticyclone (anticyclone being stronger in this case). The speed of translational motion of the pair remains approximately constant during the period of strong perturbations and afterwards during the relaxation stage when the peak values of vorticity fall in both cyclone and anticyclone. This fact indicates that the perturbations only redistribute vorticity in the vortex without entraining a significant amount of ambient (irrotational) fluid. Indeed, the impulse of the vorticity distribution $I(t) = \int_S \mathbf{x} \times \omega(\mathbf{x}, t) dS$ is equal to the total impulse $\int_0^t \int_S f(\mathbf{x}, t) dS dt$ applied by the force distribution.
function \( f(x, t) \) representing the action of the flap on the fluid (e.g., Voropayev and Afanasyev, 1994). The momentum of the vortex pair formed after the action of the external forces terminates, remains approximately constant so that the velocity of the vortex pair is constant provided that the mass of the pair is conserved.

When the anticyclonic vortex is initially in the strain field induced by its cyclonic neighbor which forms earlier, the vortex is subject to elliptical instability. The core of the anticyclone bends in a sinuous manner in the plane which corresponds to the direction of maximal strain (the direction of the initial perturbation is parallel to the plane of symmetry of the pair). The perturbed vortex tube then eventually wraps around the cyclonic vortex.

We believe that the instability observed in our experiments correspond to the so-called short-wavelength instability discussed by Leweke and Williamson (1998) for symmetric vortex pairs in nonrotating fluid rather than a long-wavelength instability (also called the Crow instability). The wavelength of these instabilities is scaled by the separation length between the vortices in the pair. Short waves have the wavelength of the order of the separation length, while the wavelength of the Crow instability is 6–8 times the separation length. Short waves are believed to be caused by mutual straining of the vortices and therefore correspond to the elliptical instability. The Crow instability is characterized by symmetric variation of the separation along the vortex tubes. The parts of the tubes where they become closer moves faster due to self-induction and, as a result “bulge” in horizontal direction. Neither the variation of the separation or any bending of the cyclonic vortex in the pair was observed in our experiments. It is therefore clear that the observed instability can be characterized as an elliptical instability modified by the background rotation.

It is interesting to discuss in more detail the effect of the development of inclined vorticity structures in the elliptically unstable vortex (see Fig. 14). The theoretical results by Le Dizes (2000) predict the dependence between the angle, \( \xi \), formed by the wavevector of instability with the vertical and the Rossby number of the flow in the following form:

\[
\cos \xi = \frac{1}{2(1 + Ro^{-1})}.
\]

The angle estimated from Fig. 14 in our experiments substituted in the above relation gives the value of the Rossby number equal to approximately 3.1. This value agrees satisfactorily well with the value of 2.7 for the initial Rossby number estimated in the experiment. The theoretical relation also predicts that \( \xi = 0 \) for \( Ro = 2 \) (see also Fig. 4 in Le Dizes, 2000). This fact agrees with our observations of vertical motions in the flows for small values of the Rossby number. Detailed investigation of the behavior of the wavevector of instability for different values of the Rossby number however requires further measurements of the vertical flow fields. This study is underway and will be reported elsewhere.

When the initial ellipticity of the vortex is varied one can expect that at low ellipticity the centrifugal instability will prevail while at high ellipticity the elliptical instability will be dominant. One can introduce the parameter \( \rho \) equal to the ratio of the length of the short axis of the vortex to its long axis to characterize the ellipticity of the vortex. The value \( \rho = 1 \) then corresponds to a circular vortex (zero ellipticity), while small values of \( \rho \) indicate high ellipticity. In the context of competition of centrifugal
and elliptical instabilities the results of Smyth and Peltier (1994) and Potylitsin and Peltier (1998, 1999, 2002) must be mentioned. Linear stability analysis by Smyth and Peltier (1994) has clearly demonstrated that in the case of an array of elliptic vortices in shear created as a result of Kelvin-Helmholtz instability, the growth rate of a mode of instability associated with centrifugal destabilization varies strongly as a function of both the Rossby number and the vertical wavenumber of the perturbation and reaches a maximum for a value of the Rossby number that is $O(1)$ when the vortices are anticyclonic. These results were confirmed and extended in (Potylitsin and Peltier, 1998). In their further analysis of Stuart vortices which are an analytical model of a vortex train in the shear flow, Potylitsin and Peltier (1999) found that the "edge" mode (centrifugal instability) is indeed a dominant mechanism of instability when the ellipticity of the vortex is low ($\rho = 0.75$). Maximum destabilization for this case was found in the vicinity of the point $Ro = 3.8$. The elliptical mode was also present and was dominant in a narrow region at high Rossby numbers $Ro = 5.9$ with the growth rate comparable to the peak growth rate of the centrifugal instability. At the other extreme, when the ellipticity of the vortex was high ($\rho = 0.35$), the elliptical mode prevailed over the entire range with maximum at the point $Ro = 3.3$. In the intermediate case ($\rho = 0.5$) the elliptical mode was dominant with maximum at $Ro = 4.4$ while centrifugal instability prevailed at the narrow region near $Ro = 10$. Direct numerical simulations of a train of Kelvin-Helmholtz vortices by Potylitsin and Peltier (2002) showed that centrifugal instability was responsible for vortex destabilization in the simulation with $Ro = 4.8$ while elliptical instability developed in the simulation with lower values of the Rossby number, $Ro = 3$.

In our experiments the competition of the centrifugal and elliptical instability was not observed because the chosen experimental set-up did not allow us to set the initial ellipticity of the vortex at our will. Moreover it is difficult to specify the initial ellipticity of the vortex in the experiment because the vortex is formed from the vortical sheet (shear layer) at the flap which subsequently wraps into the vortex. We can say however that the ellipticity changes from very high (shear layer) to intermediate when the vortex is formed ($\rho \approx 0.6$ for the anticyclone in Fig. 11a).

Finally, it should be mentioned that the observed instability was not significantly affected by viscosity in the bulk of the fluid since the typical value of the Reynolds number of the flow was large ($Re = 200 - 600$). Viscosity, however, is clearly important in the boundary layers, especially at the bottom. Ekman pumping for example causes the formation of a typical cone in the cyclonic vortex (e.g., Fig. 9). Ekman pumping also causes flow along the vortex tube of the anticyclone (Fig. 12) and is most probably the reason for tornado-like effect and formation of spiral vortices shown in Fig. 8. Formation of spiral (helical) structures is a typical phenomenon observed in swirl flows where the along-axis component of velocity is significant (e.g., Alekseenko et al., 1999).

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References


